$$W_r = \iint U(\theta, \phi)\, d\Omega \text{ watt}$$

Average Radiated Power is power per unit solid angle $= \dfrac{W_r}{4\pi}$ watt/Sr.

$$= U_{av}$$

Normalised power may be also expressed as;

$$P_n(\theta, \phi) = \frac{U(\theta, \phi)}{U_{max}(\theta, \phi)}$$

1.7 BEAM EFFICIENCY

Beam Solid Angle is due to major and minor lobes. It may be represented as;

$$\Omega_A = \Omega_m + \Omega_M$$

Ω_A = Beam solid angle

Ω_m = Minor lobe solid angle

Ω_M = Major lobe solid angle

$$1 = \frac{\Omega_M}{\Omega_A} + \frac{\Omega_m}{\Omega_A} = \xi_M + \xi_m$$

= Beam efficiency + Stray factor.

Beam efficiency is ratio of main beam area to total beam area.
Stray factor is ratio of minor beam area to total beam area.

1.8 DIRECTIVITY

Average radiated power is given by;

$$P(\theta, \phi)_{av} = \frac{1}{4\pi} \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} P(\theta, \phi) \cdot \sin\theta \cdot d\theta \cdot d\phi$$

$$= \frac{1}{4\pi} \iint_{4\pi} P(\theta, \phi) \cdot d\Omega \text{ (W-Sr}^{-1}\text{)}$$

Directivity of an antenna is ratio of maximum power density $P(\theta, \phi)_{max}$ to average value.
Directivity defines how much pattern is directional.

$$D = \frac{P(\theta, \phi)_{max}}{P(\theta, \phi)_{av.}}$$

Directivity is denoted by D. It is dimensionless

$$D \geq 1$$

$$D = \frac{P(\theta, \phi)_{max}}{\frac{1}{4\pi} \iint_{4\pi} P(\theta, \phi) d\Omega} = \frac{1}{\frac{1}{4\pi} \iint_{4\pi} \frac{P(\theta, \phi)}{P_{max}(\theta, \phi)} d\Omega}$$

$$= \frac{4\pi}{\iint_{4\pi} P_n(\theta, \phi)\, d\Omega} = \frac{4\pi}{\Omega_A} \longrightarrow \text{Beam Area}$$

Smaller beam area larger will be directivity. For an isotropic antenna radiation will be in all direction beam area is 4π.

So, Directivity of an isotropic antenna is 1.

For other antenna Directivity is greater than 1.

2.6 ANTENNA EFFICIENCY

Antenna efficiency is defined as the ratio of power radiated to the total input power fed to the antenna and it is denoted by η. Thus

$$\text{Antenna Efficiency} = \frac{\text{power radiated}}{\text{total input power}}$$

$$\eta = \frac{W_r}{W_t} = \frac{W_r}{(W_r + W_l)}$$

where

W_r = radiated power

W_t = total power fed

W_l = loss power

$$\text{Power} = I^2 R$$

$$W_r = I^2 R_r$$

where R_r = radiation resistance

$$W_t = I^2 (R_r + R_l) = E^2/(R_r + R_l)$$

Hence $$\eta = \frac{I^2 R_r}{I^2 (R_r + R_l)} = \frac{R_r}{R_r + R_l}$$

$$\eta\ \% = \left(\frac{R_r}{R_r + R_l}\right) \times 100$$

Directivity of antenna:-

It is defined as the ratio of radiation intensity in a given direction from the antenna to the avg radiation intensity over all direction.

$$D = \frac{U}{U_{avg}}$$

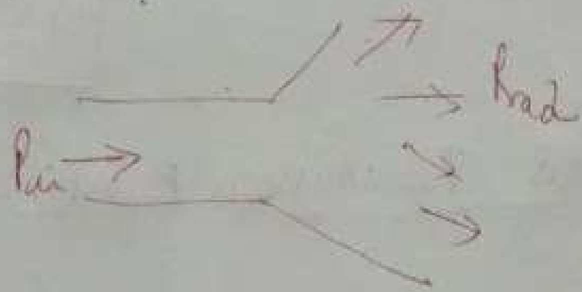
Where $U_{avg} = \frac{P_{rad}}{4\pi}$

So

$$D = \frac{4\pi U}{P_{rad}}$$

Antenna gain :- Gain of antenna is defined as the ratio of radiation intensity of test antenna to radiation intensity of refrence antenna. (Isotropic antenna)

$$G = \frac{U_{AUT}}{U_{ref}}$$



$U_{ref} = \frac{P_{in}}{4\pi}$

$$G = \frac{U_{AUT}\, 4\pi}{P_{in}}$$

Relation b/w Directivity and gain of Antenna →

We know that $D = \frac{4\pi U}{P_{rad}} \rightarrow (1)$

&

$$G = \frac{4\pi U}{P_{in}} \rightarrow (2)$$

$$(1)/(2) \Rightarrow \frac{D}{G} = \frac{\frac{4\pi U}{P_{rad}}}{\frac{4\pi U}{P_{in}}}$$

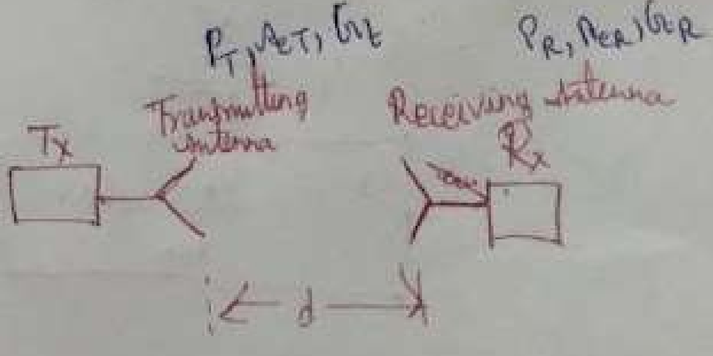
$$\frac{D}{G} = \frac{P_{in}}{P_{rad}}$$

$$\frac{G}{D} = \frac{P_{rad}}{P_{in}}$$

$$\frac{G}{D} = \eta$$ { Let $\frac{P_{rad}}{P_{in}} = \eta$ ← Antenna efficiency

$$\boxed{G = \eta D}$$

Friis Transmission Formula →



Let us consider Tx antenna as an isotropic source then power density at receiving antenna.

$$W = \frac{P_T}{A} = \frac{P_T}{4\pi d^2}$$

If G_t is the gain of Tx antenna then power density

$$W = \frac{P_T G_t}{4\pi d^2}$$

If A_{eR} is the effective aperture of Rx antenna, then received power → $P_R = W A_{eR}$

$$P_R = \left(\frac{P_T G_t}{4\pi d^2}\right) A_{eR} \rightarrow (1)$$

Gain of Tx antenna, $G_t = \frac{4\pi A_{eT}}{\lambda^2}$

Put the value of G_t in eqn (1), we get

$$P_R = \frac{P_T \, 4\pi A_{eT}}{4\pi d^2 \lambda^2} A_{eR}$$

$$P_R = \frac{P_T A_{eT}}{d^2 \lambda^2} A_{eR}$$

$$\boxed{\frac{P_R}{P_T} = \frac{A_{eR} A_{eT}}{\lambda^2 d^2}}$$