Lecture:- 17th

Date:- 26/04/2022

Method:- ③ Linear differential Equation (first order):-

$$\left[\frac{dy}{dx} + Py = Q.\right]$$

or

$$\left[\frac{dx}{dy} + Px = Q.\right]$$

where,

P, Q are either constant or function of x.

Ex:- ① $\frac{dy}{dx} + 2y = 5$

② $\frac{dy}{dx} + \sin y = \cos x$.

⊙ Rule to solve LDE:-

① $\frac{dy}{dx} + Py = Q$ · given.

write → P and Q.

② Integrating factor →

$I.f = e^{\int P \cdot dx}$

③ solution of equation ① is →

$$\boxed{y\,(I.f.) = \int Q\,(I.f)\,dx + C}$$ ☆

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Question: ① Solve $(1+x^2)\frac{dy}{dx} + 2xy - 4x^2 = 0$

Solution:
$$\frac{dy}{dx} + \frac{2xy}{1+x^2} = \frac{4x^2}{1+x^2}$$

$$\left[\frac{dy}{dx} + Py = Q\right] \text{ form.}$$

here – $P = \frac{2x}{1+x^2}$ $\quad Q = \frac{4x^2}{1+x^2}$

$$I.f. = e^{\int P.dx}$$
$$\Rightarrow e^{\int \frac{2x}{1+x^2} dx}.$$
$$\Rightarrow e^{\log(1+x^2)}$$
$$\{I.f \Rightarrow (1+x^2)\}$$

$$y.(I.f.) = \int Q(I.f) + C$$
$$y(1+x^2) = \int \frac{4x^2}{\cancel{1+x^2}} \cancel{(1+x^2)} + C$$
$$\boxed{y(1+x^2) = \frac{4x^3}{3} + C}$$

② $(1+x^2)\frac{dy}{dx} + 2xy = \cos x$

Solution:
$$\frac{dy}{dx} + \frac{2xy}{1+x^2} = \frac{\cos x}{1+x^2}$$

$$I.f. = e^{\int P.dx} \Rightarrow e^{\int \frac{2x}{1+x^2} dx} \Rightarrow e^{\log(1+x^2)} \Rightarrow (1+x^2)$$
$$[I.f. = (1+x^2)]$$

$$y(1+x^2) = \int \frac{\cos x}{\cancel{1+x^2}} \cancel{(1+x^2)} + C$$
$$\boxed{y(1+x^2) = \sin x + C} \text{ Ans.}$$

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Question (3). $(1+y^2)\,dx = (\tan^{-1}y - x)\,dy$

Solution:- $\dfrac{dx}{dy} + \dfrac{x}{1+y^2} = \dfrac{\tan^{-1}y}{1+y^2}$

here $P = \dfrac{1}{1+y^2}$, $Q = \dfrac{\tan^{-1}y}{1+y^2}$

$\Rightarrow$ I.f. $\Rightarrow e^{\int P\cdot dy}$

$\Rightarrow e^{\int \frac{1}{1+y^2} dy} \Rightarrow e^{\tan^{-1}y}$

$\Rightarrow$ [I.f. $= e^{\tan^{-1}y}$]

$\Rightarrow x\cdot(I.f.) = \int Q(I.f)\,dy + C$

$\Rightarrow x\cdot e^{\tan^{-1}y} = \int \dfrac{\tan^{-1}y}{1+y^2}\cdot e^{\tan^{-1}y}\,dy + C$

let $\tan^{-1}y = t$

$\dfrac{1}{1+y^2}\,dy = dt$

$\Rightarrow x\cdot e^t = \int t\cdot e^t\,dt + C$

$\Rightarrow x\cdot e^t = te^t - e^t + C$

$$\boxed{x\,e^{\tan^{-1}y} = \tan^{-1}y\cdot e^{\tan^{-1}y} - e^{\tan^{-1}y} + C}$$ Ans

$\circledast$ $\left[\int uv\,dv = u(v_1) - (u')v_2 + (u'')v_3 - (u''')v_4\right]$

Question:- $(x+y+1)\dfrac{dy}{dx} = 1$

Solution:- $\dfrac{dx}{dy} = x+y+1.$

$\Rightarrow \dfrac{dx}{dy} - x = y+1$

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here $P = -1$, $Q = y+1$

$$I.F. = e^{\int -1\,dy}$$

$$= e^{-y}$$

$$\Rightarrow \quad y\cdot(I\cdot F) = \int Q\,(I\cdot F)\,dy + C$$

$$\Rightarrow \quad y\,(e^{-y}) = \int (y+1)\,e^{-y}\,dy + C$$

$$ye^{-y} = (y+1)-e^{-y} - 1\,e^{-y} + C$$

$$\boxed{ye^{-y} = (-y-2)\,e^{-y} + C}$$ Ans

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