UNIT- 6TH :- Differential equation of nth order with constant coefficient

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⊙ Standard form:-

$$\boxed{a_2 \frac{d^2y}{dx^2} + a_1 \frac{dy}{dx} + a_0 = f(x)} \quad \{\text{for } n=2\}$$

$[a_i = \text{constant}]$

&

☆ complete solution = $\boxed{y = c_1 y_1 + c_2 y_2}$

⊙ Generalise form:-

$$\left[a_n \frac{d^ny}{dx^n} + a_{n-1} \frac{d^{n-1}y}{dx^{n-1}} + \cdots\cdots a_1 \frac{dy}{dx} + a_0 y = f(x)\right] \Rightarrow ★★$$

[here all a_i are constant] ☆

and,

complete solution of the above ★★ equation is →

$$\boxed{y = C.F. + P.I}$$

{∴ C.F. = Complementary function

P.I. = Particular integral

The symbol/operator D :-

$D =$ differentiation $\{D = \frac{d}{dx}\}$

$\frac{1}{D} =$ Integration.

Complementary function (C.F.) and particular integral (P.I.) :-

$$\Rightarrow a_n \frac{d^n y}{dx^n} + a_{n-1} \frac{d^{n-1} y}{dx^{n-1}} + \dots a_1 \frac{dy}{dx} + a_0 y = f(x)$$

$$\Rightarrow (a_n D^n + a_{n-1} D^{n-1} + \dots a_1 D + a_0) y = f(x)$$

$$[f(D)y = f(x)]$$

so,

$$C.F. = f(D)y = 0$$

$$P.I. = \frac{1}{f(D)} \cdot f(x)$$

by this we can find complete solution →

$$\boxed{y = C.F. + P.I.}$$

2022/5/18 23:2

Examples - $2\frac{d^2y}{dx^2} + \frac{dy}{dx} + 3y = x^2$

[$\because D = \frac{d}{dx}$
$\because f(x) = x^2$]

$\Rightarrow (2D^2 + D + 3)y = x^2$.

$[y = C.F. + P.I.]$

$C.F. = \underbrace{(2D^2 + D + 3)}_{f(D)}y = 0$

and

$P.I. = \frac{1}{2D^2+D+3} \times x^2$

⊙ Rules to find complementary functions :-

$C.F. : f(D)y = 0$

Working Steps:-

1. $D \equiv m$ {by m we have auxiliary eq.}
2. auxiliary equation $f(m) = 0$
3. find the roots of auxiliary equation $[f(m) = 0]$
4. $[m = m_1 + m_2 + m_3 (--- m_n)]$ Solution of complementary function depends on the nature of roots.

⑥ Nature of roots :-

Case-I :- If roots are real and distinct

$$m = m_1 + m_2 \quad (m_1 \neq m_2)$$

then $\left[C.F = C_1 e^{m_1 x} + C_2 e^{m_2 x} + \text{----} \right]$

Case-II :- If roots are real and ~~repeated~~ repeated →

$$m_1 \neq m_2.$$

then

$$\left[C.F = (C_1 + C_2 x) e^{m_1 x}\right] ☆$$

Ex :- If three roots are there then

$$☆\left[C.F. = (C_1 + C_2 x + C_3 x^2) e^{m_1 x}\right]☆$$

} Ans $\boxed{MCQ}$

Case-III :- If roots are imaginary →

$$m = \alpha + i\beta \qquad \{\alpha, \beta = ???\}$$

then →

$$☆\left[C.f. = e^{\alpha x}\left[C_1 \cos\beta x + C_2 \sin\beta x\right]\right]☆$$

Ex. If two times repeating then :-

$$C.f \Rightarrow e^{\alpha x}\left[(C_1 + C_2 x)\cos\beta x + (C_3 + C_4 x)\sin\beta x\right]$$

Ex :- If 10 times repeating then, → [∴ 20 constant]

$$C.f \Rightarrow e^{\alpha x}\left[(C_1 + C_2 x + C_3 x^2 + \text{---} C_{10} x^9)\cos\beta x + (C_{11} + C_{12} x + \text{-----} C_{20} x^9)\sin\beta x\right]$$