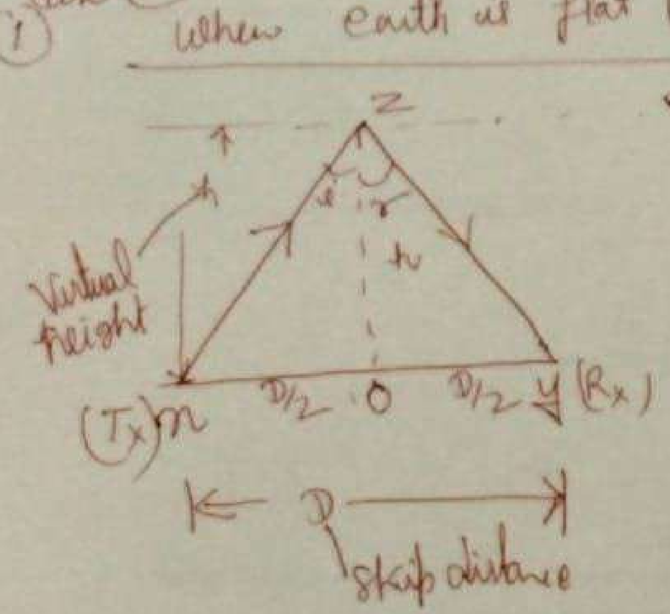
Relationship between Skip distance & MUF →

(1) case ② when earth is flat (upto 500Km)



$$\frac{f_{muf}^2}{f_c^2} = \left(1+\frac{D^2}{4h^2}\right)$$

$$f_{muf}^2 = f_c^2\left(1+\frac{D^2}{4h^2}\right)$$

$$\boxed{f_{muf} = f_c\left(1+\frac{D^2}{4h^2}\right)^{1/2}}$$

We know that → $\mu = \frac{\sin i}{\sin r} = \sqrt{1-\frac{81N}{f^2}}$ → ①

at $f = f_{muf}$, $r = 90$

So from eqn ①

$$\frac{\sin i}{\sin 90^\circ} = \sqrt{1-\frac{81N}{f_{muf}^2}}$$

$$\sin^2 i = 1-\frac{81N}{f_{muf}^2}$$

$$\frac{81N}{f_{muf}^2} = 1-\sin^2 i$$

$$\frac{81N}{f_{muf}^2} = \cos^2 i \quad ②$$

In ΔxOZ →

$$(xZ)^2 = (xO)^2 + (OZ)^2$$

$$xZ = \sqrt{\left(\frac{D}{2}\right)^2 + h^2}$$

$$\cos i = \frac{h}{xZ}$$

$$\cos i = \frac{h}{\sqrt{\left(\frac{D}{2}\right)^2+h^2}} \rightarrow ③$$

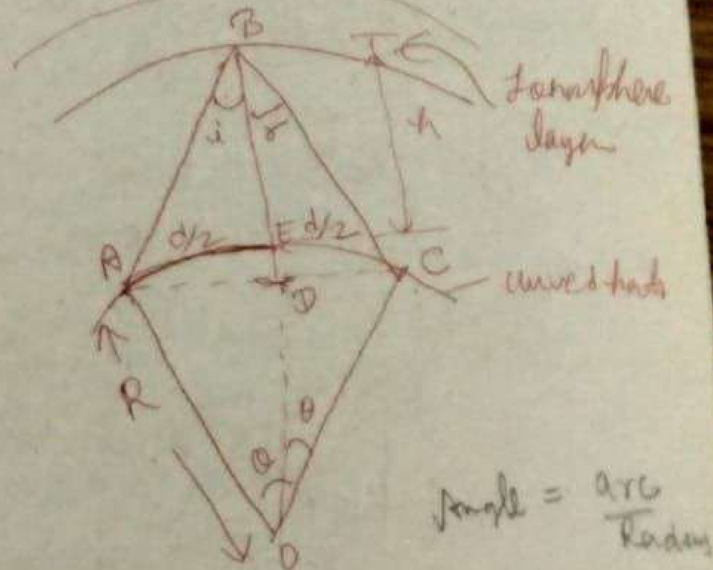
put the value of cos i from ③ into ②

$$\frac{81N}{f_{muf}^2} = \frac{h^2}{\left(\frac{D}{2}\right)^2+h^2}$$

$$\left(\frac{D}{2}\right)^2+h^2 = h^2 \times \frac{f_{muf}^2}{f_c^2}$$

Case II → (curved earth)

Let 2θ is the angle subtended by skip-distance (d) at the center of earth.



arc (d) $= 2R\theta$

Angle $2\theta = \frac{d}{R}$

Angle = arc / Radius

Let $EB = h$, $OA = OE = R$

In ΔAOD →

$$\sin\theta = \frac{AD}{R}$$

$AD = R\sin\theta$, $OD = R\cos\theta$

$$BD = OE + EB - OD$$

$$BD = R + h - R\cos\theta$$

$$AB = \sqrt{AD^2 + BD^2} \quad (\text{In } \Delta ABD)$$

$$AB = \sqrt{R^2\sin^2\theta + (R+h-R\cos\theta)^2}$$

BE + OE − OD

$$\cos i = \frac{BD}{AB} = \frac{R+h-R\cos\theta}{\sqrt{R^2\sin^2\theta + (R+h-R\cos\theta)^2}} \rightarrow (1)$$

$$\cos^2 i = \frac{(R+h-R\cos\theta)^2}{(R\sin\theta)^2 + (R+h-R\cos\theta)^2} = \frac{f_c^2}{f_{muf}^2} \rightarrow (2)$$

When skip distance is max^m $\angle OAB = 90°$

$\cos\theta = \frac{OA}{OB} = \frac{R}{(R+h)} = \left(1+\frac{h}{R}\right)^{-1}$

Since actual value of θ is very small so this relation $\cos\theta \simeq \left(1-\frac{h}{R}\right)$

can be expressed as $\rightarrow$

$\sqrt{1-\sin^2\theta} = 1-\frac{h}{R}$

$\sqrt{1-\theta^2} = 1-\frac{h}{R}$

$\boxed{\theta^2 = \frac{2h}{R}} \rightarrow (3)$

we know that $d = 2R\theta$

$d^2 = 4R^2\theta^2$

$d^2 = 4R^2 \times \frac{2h}{R}$ (from eqn 3)

$d^2 = 8hR$

$\boxed{d = \sqrt{8hR}}$

or

$h = \frac{d^2}{8R}$

we know that

$\cos\theta \simeq 1-\frac{h}{R} = 1-\frac{\frac{d^2}{8R}}{R}$

$\cos\theta \simeq 1-\frac{d^2}{8R^2} \rightarrow (4)$

from eqn (3)

$\theta^2 = \frac{2h}{R}$

$$\sin\theta \simeq \theta = \sqrt{\frac{2h}{R}}$$

$$\sin\theta \simeq \theta = \sqrt{\frac{2 \times d^2}{R \times 8R}} = \frac{d}{2R} \rightarrow (5)$$

put the value of $\sin\theta$ & $\cos\theta$ in eq^n (2) we get →

$$\cos^2 i = \frac{f_c^2}{f_{muf}^2} = \frac{R+h-R\cos\theta}{(R\sin\theta)^2 + (R+h-R\cos\theta)^2}$$

$$\frac{f_c^2}{f_{muf}^2} = \frac{\left(h+\frac{d^2}{8R}\right)^2}{\left(\frac{d^2}{4}\right) + \left(h+\frac{d^2}{8R}\right)^2}$$

$$\boxed{f_{muf} = f_c \left\{ 1 + \frac{\frac{d^2}{4}}{\left(h+\frac{d^2}{8R}\right)^2} \right\}^{1/2}}$$