Sequence & Series

Sequence :- A rule →

$f : N \longrightarrow R.$

such that →

$f(n) = a_n$ denoted as → $\underbrace{\langle f(n) \rangle \text{ or } \langle a_n \rangle}$

[presentation of sequence.]

Sequence →

$\langle f(n) \rangle = \{f(1), f(2), f(3) - - - -\}$

$= \{a_1, a_2, a_3 - - - \}$

Ex: ① $f(n) = n+2$

$\langle f(n) \rangle = \langle n+2 \rangle$

$= \{3, 4, 5, 6, 7 - - - \infty\}$

② $f(n) = n/2$

$\langle f(n) \rangle = \langle n/2 \rangle$

$= \{1/2, 2/2, 3/2, 4/2 - - - - \}$

MCQ:- Ⓐ A sequence always has. →

Ans:- Infinitely many terms.

⊙ Constant sequence :- $\left\{ \begin{array}{l} f: N \longrightarrow R \\ f(N) = k, \forall N \in R \end{array} \right\}$

⊙ Bounded above sequence :-

a sequence $\langle f(n) \rangle$ or $\langle a_n \rangle$ is said to be bounded above if there exist a real number k, such that →

$$a_n \leq k, \forall n \in N \quad [\text{here } k \text{ is upper bound.}$$

Ex :- (1) $\langle f(n) \rangle = \langle a_n \rangle = \langle \frac{1}{n} \rangle$

$$\langle 1/n \rangle = \{1, 1/2, 1/3, 1/4 \text{ ------ }\}$$

here

$\boxed{k = 1}$ $\quad \frac{1}{n} \leq 1 \; \forall \; n \in N.$

⊛ [least upper bound = smallest in all upper bounds]
(lub)

Ex → (2) $\langle f(n) \rangle = \langle a_n \rangle = \langle n \rangle$

$$= \{1, 2, 3, 4 \text{ ---- }\}$$

so, this sequence is not bounded above.

⊙ Bounded below sequence :-

$\langle a_n \rangle$ is said to be bounded below if there exists a real number l, such that →

$$l \leq a_n, \forall n \in N.$$

here $\boxed{l}$ is lower bound.

(A) [greatest lower bound = greatest in all lower bound.]
$\boxed{(glb)}$

Ex:- $\langle f(n)\rangle = \langle 1/n \rangle$ is bounded below.

$\Rightarrow \langle \frac{1}{n} \rangle = \{\frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \frac{1}{4} - - - - - - \}$

$\{l = 0, -1, -2, -3\}$ $\frac{1}{n} > 0$

[here 0 is greatest lower bound].

Bounded sequence :-

A sequence $\langle a_n \rangle$ is said to be bounded if it is bounded above as well as bounded below.

$$[\{ l \leq a_n \leq k \}, \forall\, n \in N.]$$

Ex: (1) $\frac{1}{n}$ is bounded sequence.

(2) a_n is not bounded, it is only bounded below.

(3) $\langle a_n \rangle = \langle -n \rangle$ is bounded above.

Convergent sequence :-

A sequence is said to be convergent to a real no (l) if $[\lim_{n\to\infty} a_n = l]$

Ex: $\{\langle a_n \rangle = \langle 1/n \rangle \Rightarrow \lim_{n\to\infty} \frac{1}{n} = 0$ so, convergent in 0.$\}$