Central force →

The force is always directed ~~from~~ towards or away from a fixed point and its magnitude ~~a~~ is depends on distance (r) from the origin point (O) then this force is called Central force. The particle is said to move in a Central force field, and the point (O) is reffered to as the Centre of force.

$$\boxed{\vec{F} = F(r)\,\hat{r} = F(r)\,\frac{\vec{r}}{|r|}}$$

- Here $F(r)$ is any function of distance r
- and $\hat{r} = \frac{\vec{r}}{|r|}$ is unit vector along $\vec{r}$ from the fixed point

· Force $\vec{F}$ is attractive or repulsive if $F(r) \leq 0$ or $F(r) \geq 0$ respectively.

- Central force $\boxed{F(r) = -\frac{K}{r^n}}$, Here K and n are Constant. this is known as nth power ~~of~~ law of force.

If $K > 0$ Force is attraction between the two particle.
If $K < 0$ " " repulsive " " " ".

- Central forces are Conservative forces.
- Central forces are long range forces
- ~~It~~ acts along the line Joining ~~the line joining~~ the Centre of the two bodies.

Inverse square-law force → The force between two particle is inverse square of distance between them.

$$\boxed{\vec{F} = -\frac{k}{|r|^2}\hat{r}}$$

Here k is a constant. If one particle is at the origin and the other at the position $\vec{r}$, then force acting on the second particle is given by $f = -\frac{k}{|r|^2}\hat{r}$.

Example of such forces are gravitational and electro-static forces:

$$\vec{F}_N = -\frac{G m_1 m_2}{r^2}\hat{r}$$

Newton's law of Gravitation

- Gravitational force always attractive

$$\vec{F}_c = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}\hat{r}$$

Coulomb's law

- ~~Coulombic~~ electrostatic force is attractive if both charge have opposite signs and repulsive if they have same signs

Gravitational field and potential →

The intensity of gravitational field at a point due to mass is defined as the force experienced by unit mass placed at that point. If m is the mass of a gravitating particle, the intensity of gravitational field at a distance $\vec{r}$ from it, is given as-

$$\vec{E} = -\frac{Gm}{r^2}\hat{r}$$

Here - (negative) sign shows that the direction of force is opposite to $\hat{r}$ towards the mass m. So force per unit mass is the field intensity.

→ The gravitational potential at a point is equal to potential energy per unit mass. So $\boxed{V = -\frac{GM}{r}}$ and $U = m'V = -\frac{GMm'}{r}$

and $E = -\frac{dV}{dr}$

Kepler's law. -

1- **First law** → The Path of a planet is an elliptical orbit around the sun, with the Sun at one of its foci. This law is Known as "law of ~~all~~ elliptical orbits".

2. **Second law** → The radius, drawn from the sun to a planet sweeps out equal area in equal time, i.e, its areal velocity (Area swept out by it per unit time) is constant.

3. **Third law** → its time period (its time of one complete round of the sun) is proportional to the Cube of the semi major axis of its orbit. this is Known as "harmonic law" and gives the relationship between the size of the orbit of a planet and its time of revolution.

First law →

Let us Consider a planet of mass m moving in gravitational field of sun. so The force acting on planet due to the Sun $F = G\frac{Mm}{r^2}\hat{r}$ — (1)

The force on the planet is radial and according to Newton's second law of motion it is given by

$$F = \text{mass} \times \text{radial acceleration}$$

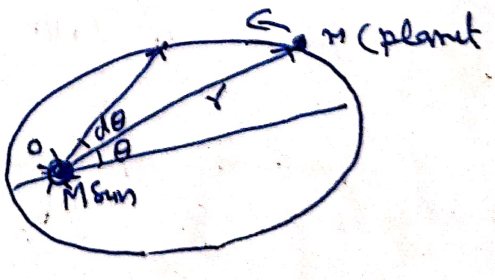
$$= m\left[\frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2\right]\hat{r} \quad \text{— (2)}$$

more accurately $\mu = \frac{mM}{m+M}$ reduced mass in place of m

But $M >> m$ so $\mu = m$

Now from eq (1) and (2)

$$\frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2 = -\frac{GM}{r^2}$$



or $$\frac{d^2r}{dt^2} - r\omega^2 = -\frac{GM}{r^2}$$

multiplying both side by r^3 we get

$$r^3\frac{d^2r}{dt^2} - r^4\omega^2 = -GMr$$

PTO

$$r^3 \frac{d^2r}{dt^2} - h^2 = -GMr \quad \text{—— (3)} \quad [\because h = \omega r^2]$$

If we put $r = \frac{1}{u}$ so that from $\omega = \frac{h}{r^2}$ we have $\left[\omega = \frac{d\theta}{dt}\right]$

$$\frac{d\theta}{dt} = hu^2$$

Now
$$\frac{dr}{dt} = \frac{d}{dt}\left(\frac{1}{u}\right) = -\frac{1}{u^2}\frac{du}{dt} = -\frac{1}{u^2}\frac{du}{d\theta}\frac{d\theta}{dt}$$

$$= -\frac{1}{u^2}\cdot\frac{du}{d\theta}\cdot hu^2 = -h\frac{du}{d\theta}$$

$$\frac{d^2r}{dt^2} = -h\frac{d^2u}{d\theta^2}\cdot\frac{d\theta}{dt} = -h^2u^2\frac{d^2u}{d\theta^2}$$

Equation (3) can be written as

$$\frac{1}{u^3}\left(-h^2u^2\frac{d^2u}{d\theta^2}\right) - h^2 = -\frac{GM}{u}$$

$$\frac{d^2u}{d\theta^2} + u = \frac{GM}{h^2}$$

or
$$\frac{d^2}{d\theta^2}\left[u - \frac{GM}{h^2}\right) + \left(u - \frac{GM}{h^2}\right) = 0 \quad \text{—— (4)}$$

This is well known eq. whose solution is $\left(\frac{GM}{h^2}\right.$ is a constant$\left.\right)$

$$u - \frac{GM}{h^2} = A\cos\theta \quad \text{or} \quad u = \frac{GM}{h^2} + A\cos\theta$$

or
$$\frac{h^2/GM}{r} = 1 + \frac{h^2A}{GM}\cos\theta \quad \text{—— (5)}$$

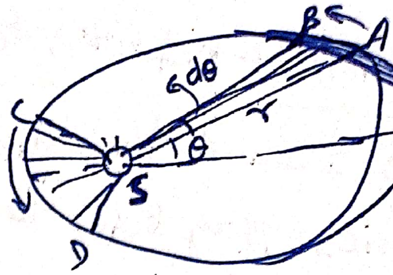
This is similar to the equation of a conic section

$$\frac{l}{r} = 1 + e\cos\theta \quad \text{—— (6)}$$

Here $e = \frac{h^2A}{GM}$ is eccentricity and $l = h^2/GM$ determine the shape of fig.

So path of planet will be ellipse

Second law → Let us consider a planet covered a distance A to B in Δt and in this time planet also covered distance C to D the by Areal velocity rule



$$\frac{\text{Area } ASB}{\Delta t} = \frac{\text{Area } CSB}{\Delta t}$$

(A) Area of $\Delta SAB = \frac{1}{2} AB \times SA = \frac{1}{2}(r\,d\theta \times r) = \frac{1}{2} r^2 d\theta$

instantaneous ~~velo~~ Areal velocity of planet

$$\frac{dA}{dt} = \frac{1}{2} r^2 \frac{d\theta}{dt} = \frac{1}{2} r^2 \omega \quad (1) \quad (\omega = \text{Angular velocity})$$

Let Angular momentum of planet around the Sun is J so

$$J = I\omega = mr^2\omega \quad \text{—(2)}$$

So from eq (1) and (2)

$$\boxed{\frac{dA}{dt} = \frac{J}{2m}} \quad \text{—(3)}$$

because ~~Area~~ Areal velocity $\frac{dA}{dt}$ of planet is constant so, Angular momentum J is also constant. due to this Angular momentum of planets ~~are~~ conserved.

"So we can say Kepler's second law is equivalent to Angular momentum conservation"

third law → we must consider the semilatus rectum l of the planet's orbit. If a and b are the semi major and semi minor axes of the elliptical path, then, $l = \frac{b^2}{a} = \frac{h^2}{GM}$ —(1)

If T is period of the planet around the sun then, $T = \frac{\text{Area of the ellipse}}{\text{areal velocity}} = \frac{\pi ab}{h/2}$

$$T^2 = \frac{4\pi^2 a^2 b^2}{h^2} = \frac{4\pi^2 a^3}{GM} \qquad \left[\because \frac{b^2}{a} = \frac{h^2}{GM}\right]$$

$$\boxed{T^2 \propto a^3}$$

Application of Kepler's law - Kepler's law describe the orbits of planets around the sun or stars around a glaxy in classical mechanics. They have been used to predict the orbits of many objects ~~much~~ such as asteroids and comets, and were pivotel in the discovery of dark matter in the milky way

Wave Particle duality :→ Interference, diffraction and Polarisation of light could be explained by wave theory of light But some phenomenon like photoelectric effect, Compton effect absorption and emission of radiation by substances could not be explained by wave theory of light. These phenomenon could be easily explained by quantum theory of light.

According to Einstein, the energy of light is concentrated into small regions. This represents the smallest quantity of energy known as "Photon". This Photon is an energy particle. Hence light shows itself wave nature as well as particle nature. This nature of light is known as dual nature and the property is known as wave-particle duality.

wave particle duality of light led Louis de Broglie in 1924

De Broglie matter waves →

According to De Broglie (1924) Matter must have dual character like light. Dual characters means that it is present in the form of Particle and shows the wave like characters. According to this concept Every moving particle has a wave packed associated with it. the wave length of such waves depends upon the momentum of the particle, $\boxed{\lambda = \frac{h}{mv} = \frac{h}{p}}$ De Broglie wavelength

"A moving particle surrounded by a wave whose wavelength depends upon the mass of the particle and its velocity. Such wave associated with the matter particle are known as matter wave or De Broglie wave."

~~Uncertainty principal~~ - Property of De Broglie wave ->

1- De Broglie waves are not electromagnetic wave
2. The matter waves are generated only when the material particles are in motion
3. The velocity of matter waves is greater than the velocity of electromagnetic wave
4. The matter ~~wave~~ generated by moving charged particles as well as by moving neutral particles.

De Broglie wave length of particle (Relativistic case)

we know $\lambda = \frac{h}{mv}$ —(1)

Here m is relativistic mass

so, $m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$ —(2)

from eq (2)

$$\frac{m_0^2}{m^2} = 1 - \frac{v^2}{c^2} \Rightarrow \frac{v^2}{c^2} = 1 - \frac{m_0^2}{m^2}$$

$$\frac{v^2}{c^2} = \frac{m^2 - m_0^2}{m^2}$$

$$m^2 v^2 = c^2 (m^2 - m_0^2)$$

$$mv = c\sqrt{m^2 - m_0^2} \quad \text{—(3)}$$

Then eq (1) will be

$$\lambda = \frac{h}{c\sqrt{m^2 - m_0^2}} = \frac{hc}{c^2(\sqrt{m^2 - m_0^2})} \quad (4)$$

Now $c^2\sqrt{m^2 - m_0^2} = \sqrt{c^4 (m - m_0)(m + m_0)}$

$$= \sqrt{[(m - m_0)c^2]\{(m - m_0)c^2 + 2m_0c^2\}}$$

But $(m - m_0)c^2 = K.E.$ is relativistic Kinetic energy.

Then

$$c^2\sqrt{m^2 - m_0^2} = \sqrt{K(K + 2m_0c^2)} \quad (5)$$

so eq (4) will be

$$\boxed{\lambda = \frac{hc}{\sqrt{K(K + 2m_0c^2)}}} \quad (6)$$