(5)- Use dynamic programming to solve

Min $z = y_1^2 + y_2^2 + y_3^2$

s.t.

$y_1 + y_2 + y_3 \geq 15$

$y_1, y_2, y_3 \geq 0$

Solution :- In this problem three decision variables are used, so it is three stage problem.

Let S_1, S_2 and S_3 are three state variables.

$S_3 = y_1 + y_2 + y_3$ ——— (1)

$S_2 = S_3 - y_3 = y_1 + y_2$ ——— (2)

$S_1 = S_2 - y_2 = y_1$ ——— (3)

The general functional eqn is

$$F_j(S_j) = \min_{y_j} \left[f_j(y_j) + F_{j-1}(S_{j-1}) \right]$$

$(1 \leq j \leq n)$

and $F_1(S_1) = f_1(y_1)$ ——— (4)

Now, $F_3(S_3) = \min_{y_3} \left[y_3^2 + F_2(S_2) \right]$ ——— (5)

$F_2(S_2) = \min_{y_2} \left[y_2^2 + F_1(S_1) \right]$ ——— (6)

$F_1(S_1) = y_1^2$

From, (5) $F_2(S_2) = \min_{y_2} \left[y_2^2 + y_1^2 \right]$

$= \min_{y_2} \left[y_2^2 + (S_2 - y_2)^2 \right]$

Find $\frac{\partial F_2(S_2)}{\partial y_2}$ lowest $2y_2 - 2(S_2 - y_2) = 0$

$y_2 = \frac{S_2}{2}$

$$F_2(S_2) = \min_{y_2}\left[\frac{S_2^2}{4} + \left(S_2 - \frac{S_2}{2}\right)^2\right]$$

$$= \min_{y_2}\left[\frac{S_2^2}{4} + \frac{S_2^2}{4}\right]$$

$$= \frac{S_2^2}{2}$$

from ②, then $F_2(S_2) = \frac{(S_3 - y_3)^2}{2}$

$$S_2 = S_3 - y_3$$

Next, $F_3(S_3) = \left[y_3^2 + \frac{(S_3 - y_3)^2}{2}\right]$

$$\frac{\partial F_3(S_3)}{\partial y_3} = 0$$

$$2y_3 + \cancel{2}\frac{(S_3 - y_3)}{\cancel{2}} = 0$$

$$3y_3 - S_3 = 0$$

$$y_3 = \frac{S_3}{3}$$

$$F_3(S_3) = \left(\frac{S_3}{3}\right)^2 + \frac{1}{2}\left(S_3 - \frac{S_3}{3}\right)^2$$

$$F_3(S_3) = \frac{S_3^2}{3}$$

$S_3 = 15$ (Assume)

Now, $S_3 = 15 \Rightarrow$ $y_3 = 5$ $y_2 = 5$

$$y_3 = \frac{S_3}{3} = \frac{15}{3} = 5$$

$$y_2 = \frac{S_2}{2} = \frac{S_3 - y_3}{2} = \frac{15 - 5}{2} = 5$$

$y_1 = 5$

$$y_1 = S_1 = S_2 - y_2$$

$$y_1 = 10 - 5 = 5$$

$$\text{Min } z = 25 + 25 + 25 = 75$$