Solution of LPP By Dynamic programming :

Q- Use dynamic programming to solve the LPP:

$$\text{Max } z = x_1 + 9x_2$$

s.t.

$$2x_1 + x_2 \leq 25$$

$$x_2 \leq 11$$

$$x_1, x_2 \geq 0$$

Solution : Let S_1, S_2 are two state variables.

The stage of dynamic programme are B_{1j} and B_{2j} for $j = 1, 2$

Now, for $j=2$

$$f_2(B_{12}, B_{22}) = \text{Max } 9x_2$$

where maximum is taken over

$$0 \leq x_2 \leq 25$$

$$0 \leq x_2 \leq 11$$

Then $f_2(B_{12}, B_{22}) = 9 \text{ Max } x_2$

$$= 9 \max\{25, 11\}$$

Since maximum value of x_2 satisfying the condition $x_2 \leq 25$ and $x_2 \leq 11$ is $\min\{25, 11\}$, that is

$x_2^* = 11$.

Now for $j=1$, $f_1(B_{11}, B_{21}) = \text{Max}\{x_1 + f_2(B_{11} - 2x_1, B_{21} - 0)\}$

where maximum of x_1 is taken by $0 \leq x_1 \leq \frac{25}{2}$

Assume $B_{11} = 25$ and $B_{21} = 11$

$f_1(25, 11) = \max\{x_1 + 9\min(25 - 2x_1, 11)\}$

$$\min(25 - 2x_1, 11) = \begin{cases} 11, & \text{for } 0 \leq x_1 \leq 7 \\ 25 - 2x_1, & \text{for } 7 \leq x_1 \leq \frac{25}{2} \end{cases}$$

$$x_1 + 9\min(25 - 2x_1, 11) = \begin{cases} x_1 + 9 \times 11, & 0 \leq x_1 \leq 7 \\ x_1 + 9(25 - 2x_1), & 7 \leq x_1 \leq \frac{25}{2} \end{cases}$$

max of $x_1 + 99$, $225 - 17x_1$ occurs at $x_1 = 7$, therefore

$f_1(25, 11) = 7 + 9\min(11, 11)$

$= 7 + 99$

$= 106$ at $x_1^* = 7$

$x_2^* = \min(25 - 2x_1^*, 11)$ as $x_1^* = 7$

$x_2^* = 11$

The optimum solution is

$x_1^* = 7$ & $x_2^* = 11$

Max $Z = 106$