B

Electric monopole – A single poin-charge can be termed as 'monopole'

" Dipole – if a monopole is displaced through a very small distance and the original monopole is replaced by an equal and opposite point charge, the system termed as a dipole.

Quadrupole – obtained by displacing a dipole through a very small distance and then replacing the original dipole by one of same magnitude but of opposite sign.

→ This concept can be continued further to obtain octapole and multipoles

→ The dipole potential varies as $\frac{1}{r^2}$

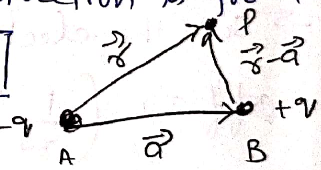
→ The Quadrupole " " " $\frac{1}{r^3}$

general $(2x)$ th multipole, the potential at large distances from the system varies as $\frac{1}{r^{x+1}}$ and the field intensity as $\frac{1}{r^{x-2}}$

Electric dipole

Two equal and opposite ~~point charges~~ separated by a small distance from a system called an electric dipole.

Electric dipole Moment :→ It is a vector quantity, whose magnitude is equal to the product of magnitude of one charges and the separation between the charges and whose direction is from negative charge; i.e $\boxed{\text{dipole moment, } \vec{p} = q.\vec{a}}$



Unit → Coulomb-meter.

Electric Potential due to an electric dipole :→

Let P is a point whose position vector relative to A is $\vec{r}$. the position vector of P relative to B will be $(\vec{r}-\vec{a})$.

The potential at P is due to $-q$ and $+q$ given as

$$V(r) = \frac{1}{4\pi\epsilon_0}\left[\frac{q}{|\vec{r}-\vec{a}|}, \frac{-q}{r}\right] \quad — (1)$$

Here a is very small than r so we expand $|(\vec{r}-\vec{a})|^{-1}$

$$|\vec{r}-\vec{a}|^{-1} = (r^2 - 2\vec{r}.\vec{a} + a^2)^{-1/2} = \frac{1}{r}\left[1 - \frac{2\vec{r}.\vec{a}}{r^2} + \frac{a^2}{r^2}\right]^{-1/2}$$

Using binomial expansion and taking only first order terms.

$$|\vec{r}-\vec{a}|^{-1} = \frac{1}{r}\left[1 + \frac{\vec{r}.\vec{a}}{r^2}\right] = \frac{1}{r} + \frac{\vec{r}.\vec{a}}{r^3} \quad — (2)$$

Putting this value in eq (1) we get

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} q\left[\frac{1}{r} + \frac{\vec{r}.\vec{a}}{r^3} - \frac{1}{r}\right] = \frac{1}{4\pi\epsilon_0} q\left(\frac{\vec{r}.\vec{a}}{r^3}\right) \quad (3) \quad [\because P = q\vec{a}]$$

$$\boxed{V(r) = \frac{1}{4\pi\epsilon_0} \cdot \frac{\vec{p}.\vec{r}}{r^3}} \quad — (4)$$

As $\nabla\left(\frac{1}{r}\right) = \frac{\vec{r}}{r^3}$ therefore eq. (4) may be expressed as

$$\boxed{V(r) = \frac{1}{4\pi\epsilon_0}\,\vec{p}\cdot\vec{\nabla}\left(\frac{1}{r}\right) = \frac{1}{4\pi\epsilon_0}\,\frac{\vec{p}\cdot\vec{r}}{r^3}} \quad -(5)$$

Electric field strength at point P found by relation

$$\vec{E}(\vec{r}) = -\,\text{grad}\,V(\vec{r})$$

using eq (4) we get

$$\vec{E}(\vec{r}) = -\,\text{grad}\left\{\frac{1}{4\pi\epsilon_0}\frac{\vec{p}\cdot\vec{r}}{r^3}\right\} = -\frac{1}{4\pi\epsilon_0}\vec{\nabla}\left(\frac{\vec{p}\cdot\vec{r}}{r^3}\right)$$

→ Now use vector identity -

$$\text{grad}(S_1 S_2) = S_1\,\text{grad}\,S_2 + S_2\,\text{grad}\,S_1$$

→ taking $S_1 = \vec{p}\cdot\vec{r}$ and $S_2 = \frac{1}{r^3}$

we get $\vec{E}(r) = -\frac{1}{4\pi\epsilon_0}\left[(\vec{p}\cdot\vec{r})\nabla\left(\frac{1}{r^3}\right) + \frac{1}{r^3}\nabla(\vec{p}\cdot\vec{r})\right]$

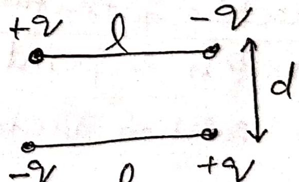
As $\vec{\nabla}(\vec{p}\cdot\vec{r}) = p$ and $\vec{\nabla}\left(\frac{1}{r^3}\right) = -3\left(\frac{\vec{r}}{r^5}\right)$ so

$$\vec{E}(r) = \frac{1}{4\pi\epsilon_0}\left[\vec{p}\cdot\vec{r}\left(-\frac{3\vec{r}}{r^5}\right) + \frac{1}{r^3}\vec{p}\right] = -\frac{1}{4\pi\epsilon_0}\left[\frac{(\vec{p}\cdot\vec{r})\vec{r}}{r^5} - \frac{\vec{p}}{r^3}\right]$$

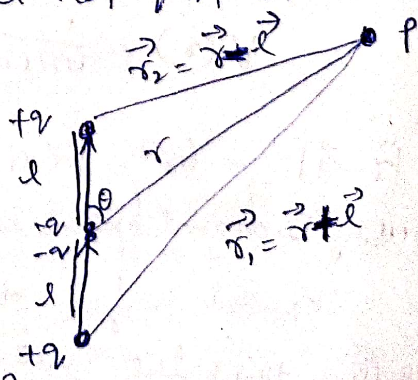
$$= \frac{1}{4\pi\epsilon_0}\cdot\frac{1}{r^3}\left[\frac{(\vec{p}\cdot\vec{r})\vec{r}}{r^2} - \vec{p}\right]$$

→ So the electric field strength due to an electric dipole fall off as $\left(\frac{1}{r^3}\right)$

Electric Quadarpole



An electric quadrupole consists of two equal and opposite dipoles that donot coincide in space, so that their electric ~~field~~ effect do not quit cancel each other at distance points

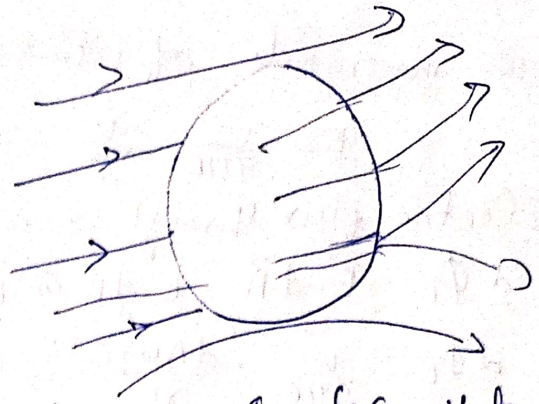


⇒ The potential due to linear Quadrupole

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\,\frac{q l^2}{r^3}(3\cos^2\theta - 1) \quad -(1)$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\left[\frac{3ql^2}{r^4}(3\cos^2\theta - 1)\hat{r} + \frac{3ql^2}{r^4}\cdot\sin 2\theta\cdot\hat{\theta}\right] \quad -(2)$$

Electric flux -



Total No. of electric lines of force passing through any surface which placed in an electric field is called electric flux.

In case of a closed surface, that is a surface that completely encloses a volume, the net flux through the surface is

$$\boxed{\phi_E = \oint \vec{E}\cdot d\vec{A}} \quad -(1)$$

where $d\vec{A}$ is the area vector of small element dA on the surface. Since $d\vec{A}$ is normal to the surface the angle between $\vec{E}$ and $d\vec{A}$ is θ and then eq (1)

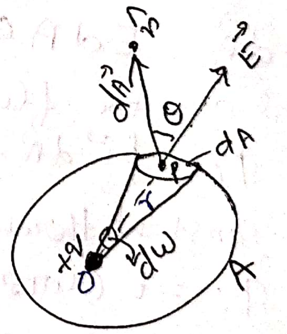
$$\boxed{\phi_E = \oint E\,dA\cos\theta} \quad -(2)$$

if plane surface normal to the electric field then, $\boxed{\phi_E = EA}$

UNIT -> Newton - m^2/Coulom, Volt-meter

Dimensional formula - $[M L^3 T^{-3} A^{-1}]$

Quantity - Scalar



Gauss's law in electrostatics

"The electric flux ϕ_E through any closed surface is equal to $\frac{1}{\epsilon_0}$ times of net charge q enclosed by the surface". $\boxed{\phi_E = \oint \vec{E}\cdot d\vec{A} = q/\epsilon_0}$

This is integral form of Gauss law.

Proof :- Let us consider a point charge +q situated at "O" inside a closed surface A. Let dA is a small area element surrounding a point P on the surface. Let OP = r. The area element may be represented by a vector $d\vec{A}$ drawn outwards along the normal to the element.

P.T.O.

The magnitude of intensity of electric field at P is

$$E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2} \quad \text{—(1)} \quad \text{where } OP = r$$

Electric flux through area element dA is

$$d\phi_E = \vec{E}\cdot d\vec{A} = E\,dA\cos\theta \quad \text{(2)} \quad [\text{Here } \theta \text{ is angle between } \vec{E} \text{ and } d\vec{A}]$$

$$d\phi_E = \frac{q}{4\pi\epsilon_0}\cdot\frac{dA\cos\theta}{r^2} \quad \text{—(3)}$$ But $\frac{dA\cos\theta}{r^2}$ is solid angle $d\omega$

Subtended by the area at the point O. therefore

$$d\phi_E = \frac{q}{4\pi\epsilon_0}\,d\omega \quad \text{—(4)}$$

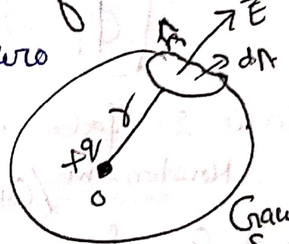
Total flux ϕ_E through entire surface A is

$$\phi_E = \frac{q}{4\pi\epsilon_0}\oint d\omega \quad \text{—(5)}$$ But $\oint d\omega = 4\pi$, the solid angle

subtended by the entire closed surface A at the point O. So

$$\boxed{\phi_E = \frac{q}{\epsilon_0}} \quad \text{— (6)}$$ Hence proove

Electric field due to point charge [Deduction of Coulomb's law from Gauss's law]



from fig angle between $\vec{E}$ and $d\vec{A}$ will be zero due to same direction. so

$$\vec{E}\cdot d\vec{A} = E\,dA\cos 0 = E\,dA \quad \text{—(1)}$$

Total electric flux leaving the Gaussian Surface is

$$\phi_E = \oint \vec{E}\cdot d\vec{A} = E\oint dA \quad \text{— (2)}$$

E is constant throughout the surface. But $\oint dA = 4\pi r^2$ (area of the sphere)

$$\phi_E = E(4\pi r^2) \quad \text{—(3)}$$ using Gauss theorem.

$$\frac{q}{\epsilon_0} = E(4\pi r^2) \Rightarrow E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2} \quad \text{—(4)}$$

If we put a test charge q_0 at that point then the magnitude of force experienced by q_0 will be -

$$\boxed{F = q_0 E = \frac{1}{4\pi\epsilon_0}\frac{q q_0}{r^2}} \quad \text{—(5)}$$

Differential form of Gauss's law →

~~According integral form of Gauss's law.~~ $\oint_S \vec{E}\cdot d\vec{A} = \frac{q}{\epsilon_0}$ —(1)

Let ρ is volume charge density then net charge $q = \int_V \rho\,dV$ —(2)

So eq (1) will be $\oint_S \vec{E}\cdot d\vec{A} = \frac{1}{\epsilon_0}\int_V \rho\,dV$ —(3)

According to Gauss divergen theorem $\oint_S \vec{E}\cdot d\vec{A} = \int_V (\vec{\nabla}\cdot\vec{E})\,dV$ —(4)

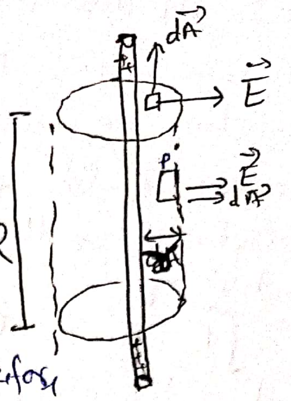
Comparing (3) and (4) $\oint(\vec{\nabla}\cdot\vec{E})\,dV = \frac{1}{\epsilon_0}\int \rho\,dV$ or $\int(\vec{\nabla}\cdot\vec{E})\,dV = \int\left(\frac{\rho}{\epsilon_0}\right)dV$ (5)

This can true for any arbitrary volume V only when

$$\boxed{\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0} \text{ or } \operatorname{div}\vec{E} = \frac{\rho}{\epsilon_0}} \quad \text{— (6)}$$

Application of Gauss law

① Electric field due to an infinite line of charge



Let linear charge density of thin wire or thin long charged rod λ C/m. Let P is a point distance r meter from the wire at which the $\vec{E}$ is required.
Draw a Co-axial Gaussian cylindrical surface of length l through the point P. E will be same at all points on this surface and directed radially outwards. therefore

$$\vec{E}\cdot d\vec{A} = E\,dA\cos 0 = E\,dA$$

So electric flux through the Gaussian surface is

$$\phi_E = \int_A \vec{E}\cdot d\vec{A} = \int_A E\,dA = E\int_A dA = E(2\pi r l) \quad -①$$

Flux through the plane ends of the surface is zero because $E \perp dA$

So total flux through Gaussian surface.

$$\phi_E = E(2\pi r l) \quad -②$$

According to Gauss law, $\phi_E = \frac{q}{\epsilon_0}$, where $q = \lambda l$ so that

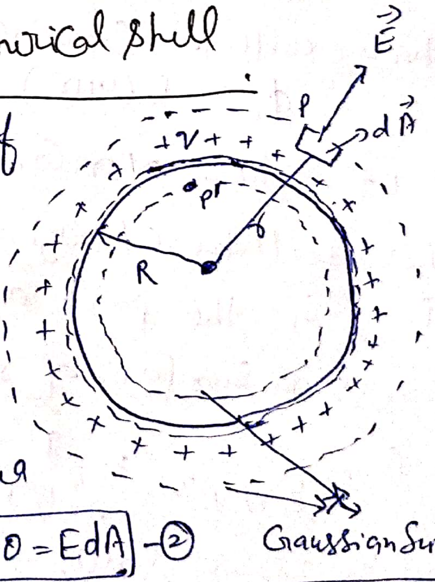
$$\phi_E = \frac{\lambda l}{\epsilon_0} \quad ——③$$

Comparing ② and ③ we get

$$E(2\pi r l) = \frac{\lambda l}{\epsilon_0} \quad \text{or} \quad E = \frac{\lambda}{2\pi\epsilon_0 r} \quad \text{or} \quad \boxed{\vec{E} = \frac{1}{2\pi\epsilon_0}\frac{\lambda}{r}\hat{r}} \quad -④$$

$$\text{or} \quad \boxed{\vec{E} = \frac{1}{4\pi\epsilon_0}\cdot\frac{2\lambda}{r}\hat{r}} \quad -⑤$$

② Electric field due to a charged Spherical shell



Let us consider a spherical shell of radius R having surface charge density σ then $\boxed{q = 4\pi R^2\sigma}$ $-①$

Case-1 → ($r>R$) At external point (outside the charge shell)

from fig the angle between $\vec{E}$ and $d\vec{A}$ at point P zero. electric flux through area element dA is, $\boxed{d\phi_E = \vec{E}\cdot d\vec{A} = E\,dA\cos 0 = E\,dA}$ $-②$

flux through entire area element dA is $\boxed{\phi_E = \oint E\,dA = E(4\pi r^2)}$ $-③$

By Gauss theorem $\phi_E = \frac{q}{\epsilon_0} = \frac{4\pi R^2\sigma}{\epsilon_0}$ ——④ [use eq ①]

from eq ③ and ④ $E(4\pi r^2) = \frac{4\pi R^2\sigma}{\epsilon_0}$.

$$\boxed{E = \frac{\sigma}{\epsilon_0}\frac{R^2}{r^2}} \quad -④$$

Case-2 → At surface ($r=R$) → $\boxed{E = \frac{\sigma}{\epsilon_0}}$ $-⑤$

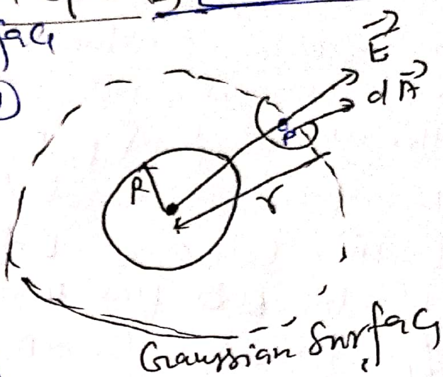
Case 3. At internal point (Inside the shell) ($r<R$)

Gaussian surface through P' does not enclose any charge, so according to Gauss theorem $\boxed{\phi_E = E(4\pi r^2) = 0 \text{ or } E=0}$ — (6)

(3) Electric field of a uniformly charged sphere, $\boxed{\text{Case-1} \rightarrow r>R}$

Total electric flux through the Gaussian surface of radius r.

$\phi_E = \oint \vec{E}\cdot d\vec{A} = \vec{E}\oint_s dA = E(4\pi r^2)$ — (1)



According to Gauss theorem. $\phi_E = \frac{q}{\epsilon_0}$ — (2)

from (1) and (2) $\boxed{E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}}$ — (3)

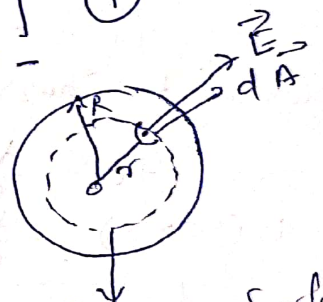
So electric field strength at any point outside a charged sphere is the same as if the whole charge were concentrated at the centre

Case-2 $r=R$. eq (3) will be $\boxed{E = \frac{1}{4\pi\epsilon_0}\frac{q}{R^2}}$ — (4)

Case-3 ($r<R$) when point lies inside the sphere -

flux through Gaussian Surface.

$\phi_E = E(4\pi r^2)$ — (5)



There arises two cases.

(A) if sphere is conducting →

Charge within Gaussian surface will be zero then

$\phi_E = E(4\pi r^2) = 0$ or $\boxed{E=0}$ — (5A)

(B) if sphere non conducting →

There will be charge distribution through whole volume if q' is the part of q which is inclosed within the Gaussian surface of radius (r), then volume density

$\rho = \frac{q}{\frac{4}{3}\pi R^3} = \frac{q'}{\frac{4}{3}\pi r^3}$ so from this $q' = q\left(\frac{r}{R}\right)^3$ — (6)

and by Gauss law, flux through Gaussian Surface of radius r. $\phi_E = \frac{q'}{\epsilon_0}$ using (5) $E(4\pi r^2) = \frac{q'}{\epsilon_0}$

or $\boxed{E = \frac{1}{4\pi\epsilon_0}\frac{q'}{r^2}}$ — (7) Putting the value of q' from (6)

$\boxed{E = \frac{1}{4\pi\epsilon_0}\frac{qr}{R^3}}$ — (8)

→ So E due to a uniformly charged sphere of an internal point is proportional to distance r of the point from centre