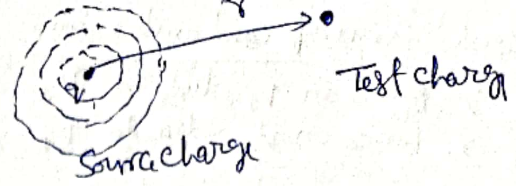
Electrostatics →



In electrostatics all the Source charge are stationary and test charge may be moving.

Quantisation of charge [fundamental charge].

charge is created by transfer of electron, so net charge on a body is always an integral multiple of charge on an electron. Charge in a body is produced due to excess or deficiency of electron. Electron can not be divided into further smaller parts. Therefore charge on a body in integral multiple of the charge on electron.

The magnitude of charge on an electron is called fundamental charge or elementary charge. Its value is 1.6×10^{-19} Coulomb and denoted by e. Therefore we may say "Any physically existing charge is always an integral multiple of fundamental charge e".
This is called the principle of atomicity of charge or principle of quantization of charge.

if q is charge on a body and n is positive or negative integral number, then - $\boxed{q = \pm ne}$ is also called Quantum of charge.

Coulomb's law → The force of attraction or repulsion between two point charges is directly proportional to the product of the charge and inversely proportional to the square of distance between them. The direction of this force is along the line joining the two charges. This law is called Coulomb's law (inverse square law).

If two point charge q_1 and q_2 separated by a distance r, then the force acting between them is -

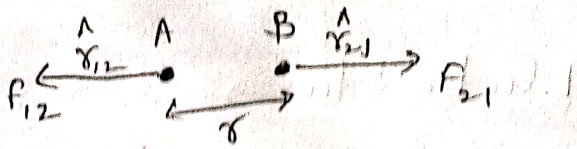
$$F \propto q_1 q_2$$

$$\propto \frac{1}{r^2}$$

$$\boxed{F = k\frac{q_1 q_2}{r^2}} \quad -①$$

Here k is proportionality constant $\frac{1}{4\pi\epsilon_0 K} \approx 9 \times 10^9 \frac{N\text{-}m^2}{C^2}$
and ϵ_0 is universal constant, called as permittivity of vacuum or free space and K is dielectric constant
$\epsilon_0 = 8.85 \times 10^{-12}\ C^2/N\text{-}m^2$

Vector form of Coulomb's law



$F_{12} \rightarrow$ Force on q_1 due to q_2

$F_{21} \rightarrow$ force on q_2 due to q_1

$\vec{F}_{21} = \frac{1}{4\pi\epsilon_0 K} \frac{q_1 q_2}{r^2} \hat{r}_{21}$ — ① . But $\hat{r}_{21} = \frac{\vec{r}_{21}}{|r|}$ so

$\vec{F}_{21} = \frac{1}{4\pi\epsilon_0 K} \frac{q_1 q_2}{r^3} \vec{r}_{21}$ — ② Similarly $\vec{F}_{12} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^3} \vec{r}_{12}$ — ③

Simply $\vec{r}_{12} = -\vec{r}_{21}$ so from eq ③ $\vec{F}_{12} = -\frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^3} \vec{r}_{21}$ — ④

Comparing ② and ④ $\boxed{\vec{F}_{21} = -\vec{F}_{12}}$ — ⑤

Coulomb -

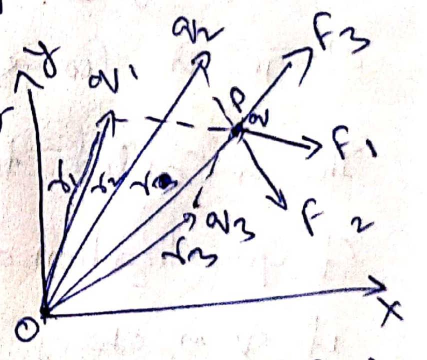
1- 1 Coulomb is a charge which when placed at a distance of 1 meter from an equal and similar charge in vaccum [air, K=1] will repel it with force 9×10^9 Newton.

2- 1 Coulomb is the charge that is produced by removal of 6.25×10^{18} electrons from a neutral body. [$q = ne = 6.25 \times 10^{18}$]

3- 1 Coulomb is the charge which when flowing in a conductor for 1 seconds causes a current of 1 amp. [$q = i.t$]

Principle of superposition →

If the system ~~contains a number~~ of interacting charges, then the force on a given charge is equal to the vector sum of the forces exerted on it by all remaining charges.



So force on point P due to $q_1, q_2 \cdots q_n$ charges.

in fig $\vec{F} = \vec{F}_1 + \vec{F}_2 + \cdots \vec{F}_n$

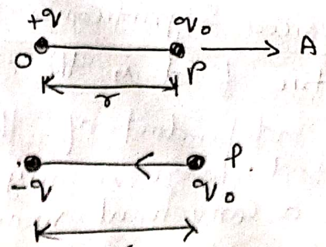
$$= \sum_{i=1}^{n} \vec{F}_i = \sum_{i=1}^{n} \frac{1}{4\pi\epsilon_0} \frac{q q_i}{|r - r_i|^3} (\vec{r} - \vec{r}_i) = \frac{1}{4\pi\epsilon_0} q \sum_{i=1}^{n} \frac{q_i (\vec{r} - \vec{r}_i)}{|r - r_i|^3}$$

Electric field Strength [Electric field intensity]

The region in which a charge experience a force is called electric field. The electric field strength at any point in any ~~point~~ electric fields is a vector quantity, whose magnitude is equal to force acting per unit positive test charge and whose direction is along the direction of force.

$\vec{E} = \frac{\vec{F}}{q_0}$	Unit → Newton/Coulomb or Volt/meter	Dimensional formula $[MLT^{-3}A^{-1}]$

(A) $\vec{E}$ due to point charge



For electric field strength at a point in the electric field we place a very small (infinitesimal) positive charge (q_0) at that point. This charge is very small so it called Test charge and due to smallness it does not cause of any change in initial electric field.

→ According to Coulombs law, for +q charge

$$\vec{F} = \frac{1}{4\pi\epsilon_0}\frac{q q_0}{r^2}\hat{r} \quad [P \to A]$$

then

$$\vec{E} = \frac{\vec{F}}{q_0} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r} \quad (P \to A) \quad \text{— (1)}$$

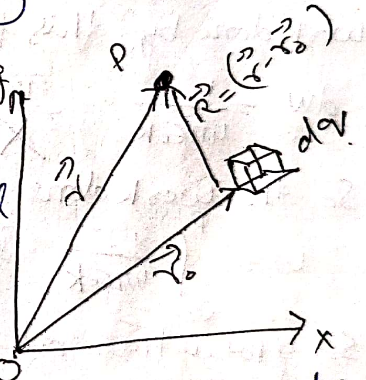
→ If charge is $-q$ then

$$\vec{E} = \frac{\vec{F}}{q_0} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r} \quad (P \to O) \quad \text{— (2)}$$

Here $\hat{r}$ is unit vector along vector $\vec{r}$, so

$$\boxed{\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{|r|^3}\vec{r}} \quad \text{— (3)}$$

(B) Due to continuous charge distribution



Continuous charge distribution may be supposed to be formed of a large number of very small charge element.

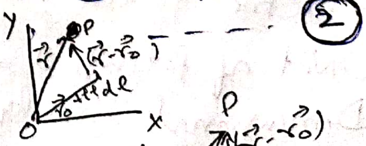
Consider a small element of charge dq, if charge dq is located at position $\vec{r}_0$ and then the point P under consideration located at ~~point~~ position $\vec{r}$, then the distance of point P from this charge element is R. The electric field strength at P due to charge element

$$d\vec{E} = \frac{dq}{4\pi\epsilon_0 R^2}\hat{R} \quad \text{--- (1)}$$

Total electric field at P for whole charge distribution.

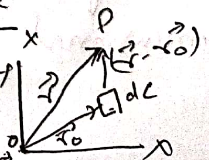
$$\vec{E} = \frac{1}{4\pi\epsilon_0}\int\frac{dq}{R^2}\hat{R} = \frac{1}{4\pi\epsilon_0}\int\frac{dq}{R^3}\vec{R} = \frac{1}{4\pi\epsilon_0}\int\frac{dq\,(\vec{r}-\vec{r}_0)}{|\vec{r}-\vec{r}_0|^3} \quad \text{(2)}$$

→ (i) Due to line charge → $dq = \lambda\, dl$, so eq (2) will be



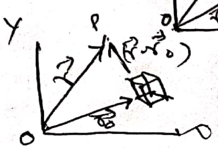
$$\boxed{\vec{E} = \frac{1}{4\pi\epsilon_0}\int\frac{\lambda\, dl}{|\vec{r}-\vec{r}_0|^3}(\vec{r}-\vec{r}_0)} \quad \text{— (A)}$$

(ii) Due to surface charge distribution → $dq = \sigma\, ds$



$$\boxed{E = \frac{1}{4\pi\epsilon_0}\int\frac{\sigma\, ds}{|\vec{r}-\vec{r}_0|^3}(\vec{r}-\vec{r}_0)} \quad \text{— (B)}$$

(iii) Due to ~~volume charge~~ distribution → $dq = \rho\, dv$



$$\boxed{E = \frac{1}{4\pi\epsilon_0}\int\frac{\rho\, dv}{|\vec{r}-\vec{r}_0|^3}(\vec{r}-\vec{r}_0)} \quad \text{— (C)}$$

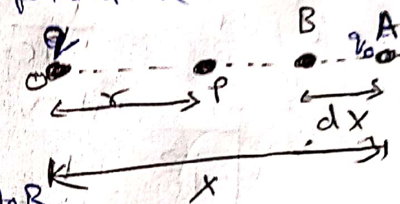
Physical Significance of electric field

→ Electric field in the characteristic of the charges of system and independent of the test charge.
→ The test charge is only introduced for measurement of electric field in a convenient manner.
→ The true physical significance of electric field ~~electric field~~ appears only when we keep in view that electrostatic interaction is only a part of general fundamental force, known as electromagnetic interaction.

Electrostatic Potential

The workdone for Bring a unit positive charge from infinity to any point into electric field is called potential at that point.
Potential gives the direction in which a free positive charge will tend to move. If a unit charge is left free, it will move from higher potential to lower potential.

→ Force at q_0 when placed at A



$$F = \frac{1}{4\pi\epsilon_0 K} \frac{qq_0}{x^2} \quad [\text{Along } \vec{on}]$$

work done by this force in moving q_0 from A to B (distance $-dx$)

$$dw = \frac{1}{4\pi\epsilon_0 k} \cdot \frac{qq_0}{x^2}(-dx)$$

So the workdone in bringing charge q_0 from ∞ to P

$$W = \int_{\infty}^{P} \frac{1}{4\pi\epsilon_0 k} \frac{qq_0}{x^2} dx = \frac{1}{4\pi\epsilon_0 k} \cdot \frac{qq_0}{r}$$

So a potential at P [workdone is moving unit charge from ∞ to P]

$$\boxed{V = \frac{W}{q_0} = \frac{1}{4\pi\epsilon_0 k} \frac{q}{r}} \quad \text{for air } K=1 \quad \boxed{V = \frac{1}{4\pi\epsilon_0} \frac{q}{r}}$$

Electric field and potential difference

Electric field intensity is equal to the negative gradient of potential.

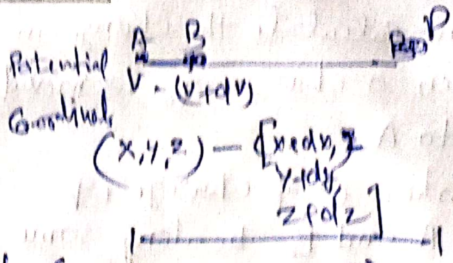
$$\boxed{E = -\frac{dV}{dx} \text{ or } dV = -E dx}$$

Negative sign signifies that potential decreases in direction of electric field.

UNit of Potential → Volt, Joule/coulomb, Newton-meter/Coulomb
Dimensional formulas $[ML^2T^{-3}A^{-1}]$, Quantity → Scalar

Electric field from potential →

The small displacement from A to B



$$d\vec{r} = dx\hat{i} + dy\hat{j} + dz\hat{k} \quad (1)$$

As V is function of co-ordinate (x, y, z)
so small change dV is ~~function~~ the value of potential V,
Corresponding to small displacement $d\vec{r}$ from A to B is given as

$$dV = \frac{\partial V}{\partial x}dx + \frac{\partial V}{\partial y}dy + \frac{\partial V}{\partial z}dz \quad — (2)$$

$$= \left[\hat{i}\frac{\partial V}{\partial x} + \hat{j}\frac{\partial V}{\partial y} + \hat{k}\frac{\partial V}{\partial z}\right]\cdot\left[dx\hat{i} + dy\hat{j} + dz\hat{k}\right]$$

$$= \left[\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right]V \cdot \left[dx\hat{i} + dy\hat{j} + dz\hat{k}\right]$$

$$dV = \vec{\nabla}V \cdot d\vec{r} \quad — (3)$$

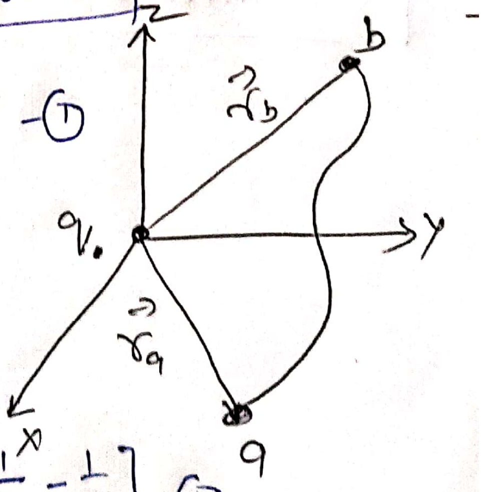
But we know $dV = -\vec{E}\cdot d\vec{r}$ — (4)

So from (3) and (4)

$$-\vec{E}\cdot d\vec{r} = \vec{\nabla}V\cdot d\vec{r} \quad \text{or} \quad \vec{E}\cdot d\vec{r} = (-\vec{\nabla}V)\cdot d\vec{r}$$

$$\text{or} \quad \boxed{\vec{E} = -\vec{\nabla}V} \quad \text{or} \quad \boxed{\vec{E} = -\text{grad}\,V} \quad — (5)$$

Curl of electric field $[\vec{\nabla}\times\vec{E}]$



The electric field for q is, $\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$ — (1)

So from fig. for spherical co-ordinate

$$d\vec{r} = dr\hat{r} + r\,d\theta\,\hat{\theta} + r\sin\theta\,d\phi\,\hat{\phi} \quad — (2)$$

and so — $\vec{E}\cdot d\vec{l} = \vec{E}\cdot d\vec{r}$ — (3)

$$\oint_{r_a}^{r_b}\vec{E}\cdot d\vec{l} = \int_{r_a}^{r_b}\frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}\cdot(dr\hat{r} + r\,d\theta\,\hat{\theta} + r\sin\theta\,d\phi\,\hat{\phi})$$

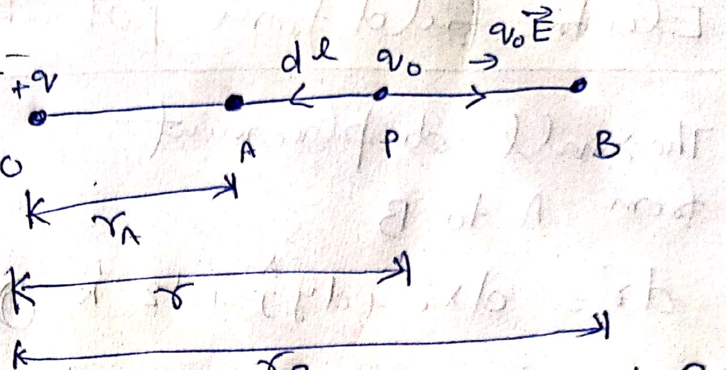
$$\int_{r_a}^{r_b}\vec{E}\cdot d\vec{l} = \frac{q}{4\pi\epsilon_0}\int_{r_a}^{r_b}\frac{dr}{r^2} = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{r}\right]_{r_a}^{r_b} = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{r_b} - \frac{1}{r_a}\right] \quad — (4)$$

for closed path $r_a = r_b$ so that, $\int_{r_a}^{r_b}\vec{E}\cdot d\vec{l} = 0$ — (5)

By Stoke's theorem $\oint\vec{E}\cdot d\vec{l} = \iint(\vec{\nabla}\times\vec{E})\,ds$ — (6)

from (5) and (6) $\boxed{\vec{\nabla}\times\vec{E} = 0 \text{ or } \text{Curl}\,\vec{E} = 0}$ — (7)

Work and Energy in Electrostatics



Let us now calculate the change in P.E. when a charge q_0 is moved from B to A.

Let the center O of charge $+q$ and point A and B be along same straight line. If P is any point on the line between A and B and a test charge q_0 is placed at P, then electric field will exert a force $\boxed{\vec{F} = q_0 \vec{E}}$ on the test charge q_0.

An external force $\boxed{\vec{F_e} = -q_0 \vec{E}}$ is required on the test charge to keep it in equilibrium.

So workdone in small displacement $d\vec{l}$ from P towards A.

$$dW = \vec{F_e} \cdot d\vec{l} = -q_0 \vec{E} \cdot d\vec{l} = -q_0 E\, dl \cos 180^\circ \quad [\because \vec{E} \text{ and } d\vec{l} \text{ are in opposite directions}]$$

$$= -q_0 E\, dl\, (-1) = q_0 E\, dl$$

But $dl = -dr$ ($\because dl$ is measured in the direction of decreasing r)

$$dW = q_0 E(-dr) = -\frac{q q_0}{4\pi r^2} dr \qquad \left[\because E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}\right]$$

Total workdone, taking test charge q_0 from $\vec{B}$ to $\vec{A}$ is

$$W = \int_{r_B}^{r_A} dW = \int_{r_B}^{r_A} -\frac{1}{4\pi\epsilon_0} \frac{q q_0}{4\pi\epsilon_0 r^2} dr = -\frac{q q_0}{4\pi\epsilon_0} \int_{r_B}^{r_A} \frac{1}{r^2} dr = -\frac{q q_0}{4\pi\epsilon_0}\left[-\frac{1}{r}\right]_{r_B}^{r_A}$$

$$\boxed{W = \frac{q q_0}{4\pi\epsilon_0}\left[\frac{1}{r_A} - \frac{1}{r_B}\right] = (PE)_A - (PE_B)}$$