Experiment No. 8

KATER'S (REVERSIBLE) PENDUL

Object: To Determine the acceleration due to earth gravity with Kater's pendulum

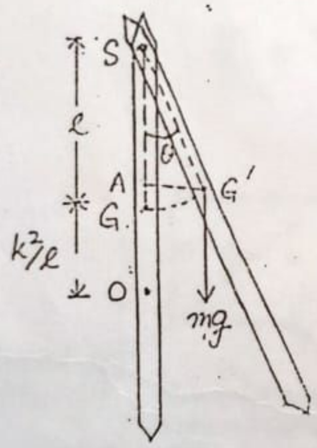
Apparatus: Kater's Pendulum watc

Construction -- **Construction:** Kater's pendulum is a long metallic rod having two adjustable knife edges near two ends and two cylinders – large and small, that can be made to slide along the rod and clamped in any position. Finer adjustments can be made with the wooden blocks. The pendulum can be made to oscillate about either of the two knife edges.

Theory: Let S be the centre of suspension & G the centre of gravity of the pendulum.(see fig. 1). If the pendulum is displaced through an angle θ, its centre of gravity (CG) G takes a position G′. The weight of the pendulum mg and its reaction at the support constitute a couple given by

$$C = -mg(G'A) = -mgl\sin\theta \qquad \ldots\ (1)$$

Where SG = L



(figure 1)

This couple produces an angular acceleration in the body. If I is the a moment of inertia of the body about the axis of oscillation through S, then

$$C = I\frac{d^2\theta}{dt^2}$$

Therefore,

$$I\frac{d^2\theta}{dt^2} = -mgl\sin\theta \qquad \ldots\ (2)$$

If θ is small, this equation represents a simple harmonic motion with time period

$$T = 2\pi\sqrt{I/mgl} \qquad \ldots\ (3)$$

If I_g is the moment of inertia about a parallel axis through CG. Then

$$I = I_g + ml^2$$
$$mk^2 + ml^2$$

where K is the radius of gyration about CG. Substituting this in (3) we get,

$$T = 2\pi\sqrt{\frac{\ell + k^2/\ell}{g}} = 2\pi\sqrt{\frac{L}{g}} \qquad \ldots\ (4)$$

where $L = l + k^2/l$ is the length of an equivalent simple pendulum. If we take O on the line SG produced such that $SO = l + k^2/l$ then O is called the centre of oscillation. It can be seen that for a given configuration of the Kater's pendulum, the time period about S and O is the same (check it)

Now let the time periods about the two knife edges be T_1 and T_2 respectively and l_1 and l_2 be their respective distances from the CG.

Then

$$T_1 = 2\pi\sqrt{(k^2+l_1^2)/l_1 g} \quad \dots (5)$$

And

$$T_2 = 2\pi\sqrt{(k^2+l_2^2)/l_2 g} \quad \dots (6)$$

Squaring and rearranging (5) and (6)

$$\frac{4\pi^2}{g} = \frac{T_1^2+T_2^2}{2(l_1+l_2)} + \frac{T_1^2-T_2^2}{2(l_1-l_2)} \quad \dots (7)$$

or

If $T_1 \approx T_2$ i.e., if the two knife edges are so placed that one of them lies at the centre of suspension while the other is very nearly at the centre of oscillation, then the second term on the right may be neglected and we have

$$\frac{4\pi^2}{g} = \frac{T_1^2 + T_2^2}{2(l_1+l_2)}$$

or

$$\boxed{g = \frac{4\pi^2(l_1 + l_2)}{(T_1^2 + T_2^2)/2}} \quad \dots (8)$$

g

Procedure:

(A) (i) The knife edges are fixed firmly near the ends of the rod and the distance between the knife edges ($l_1 + l_2$) is noted.

(ii) One of the adjustable weights (the larger one) is fixed and the distance of the other (which is to be adjusted by sliding) is noted from any one knife edge. Time for about 50 to 100 oscillations about one knife edge is recorded. Time is also recorded by inverting the pendulum so that it oscillates about the other knife edge.

(iii) The adjustable weight is now slowly moved in one direction only and the above procedure is repeated till the time periods T_1 and T_2 about the two knife edges are nearly equal. The value of g can be calculated then using formula (8).

(B) Use a meter scale to find the C.G. after removing the small cylinders and keeping the large cylinder fixed in the middle. Then record the time periods of the pendulum at different centres of suspension by varying their distances from the C.G. Repeat the same by inverting the rod . Plot a curve between time period T vs. distance of the point of suspension from the CG. Locate on the curve the centre of suspension & oscillation at a given T. Calculate the value of g from this.

Also record the point of suspension for minimum and maximum time period and the value of radius of gyration about the C.G. for this configuration from the curve.

Question : Discuss the sources of error and corrections for them.