8/5/22.

Solution of linear differential eqⁿ with constant coefficients :

Standard form:

The equation of a form

$$a_0\frac{d^n y}{dx^n} + a_1\frac{d^{n-1}y}{dx^{n-1}} + a_2\frac{d^{n-2}y}{dx^{n-2}} + \cdots + a_n y = Q.$$

Where $a_0, a_1, \cdots a_n$ are all constants and Q is a function of x alone is called a linear differential eqⁿ of order n, with constant coefficient.

Complimentary function (C.F.)
Particular integral (P.I.)

General Solution -

$$\boxed{y = C.F + P.I.}$$

Q Solve :

$$\frac{dy}{dx} + y = e^x$$

Sol. $Dy + y = e^x$

$(D+1)y = e^x$

Auxiliary Eqⁿ - put $D = m$

$m + 1 = 0$

$m = -1$

$f(D)y = Q$

$$\boxed{P.I. = \frac{1}{f(D)} Q}$$

Case I: If roots of the auxiliary equⁿ are ~~different~~ different i.e.

$m = m_1, m_2, m_3$ - Then.

$$C.F. = C_1 e^{m_1 x} + C_2 e^{m_2 x} + C_3 e^{m_3 x} + \cdots\cdots$$

Case II: If roots of the auxiliary equⁿ are different i.e.,

$m = m_1, m_2, m_4$, then

$$C.F. = (C_1 + C_2 + C_3 x^2) e^{m_1 x}$$

Case III: If roots of the auxiliary equⁿ are ~~of the~~ imaginary. i.e.,

$m = \alpha \pm i\beta$.

$$C.F. = C_1 e^{\alpha x} \cos(\beta x + C_2)$$
$$= C_1 e^{\alpha x} \sin(\beta x + C_2)$$
$$= e^{\alpha x}(C_1 \cos\beta x + C_2 \sin\beta x) \quad \text{— preferred.}$$

Case IV: If roots of the auxilary equⁿ are irrational - i.e, $m = \alpha \pm \sqrt{\beta}$

$$C.F = C_1 e^{\alpha x} \cosh(\sqrt{\beta} x + C_2)$$

$$= C_1 e^{\alpha x} \sinh(\sqrt{\beta} x + C_2)$$
$$= e^{\alpha x}(C_1 \cosh\sqrt{\beta} x + C_2 \sinh\sqrt{\beta} x) \quad \text{— preferred.}$$

Note : If $Q = 0$
then $P.I = 0$

Q. Solve : $(D^2 - 2D + 4)y = 0$

Ans - Auxiliary equⁿ is -

$(m^2 - 2m + 4) = 0$

$\frac{b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{2 \pm \sqrt{4 - 16}}{2}$

$= 2 \pm i\sqrt{12}$

$= \frac{2 \pm 2i\sqrt{3}}{2}$

$= 1 \pm i\sqrt{3}$

$C.F. = e^x (C_1 \cos\sqrt{3}x + C_2 \sin\sqrt{3}x)$

$P.I = 0 \quad [\text{since } Q = 0]$

$y = e^x (C_1 \cos\sqrt{3}x + C_2 \sin\sqrt{3}x)$

Q. Solve : $\left(\frac{d^3y}{dx^3} - 7\frac{dy}{dx} - 6y\right) = 0$

Sol $\Rightarrow D^3y - 7Dy - 6y = 0$

$\Rightarrow (m^3 - 7m - 6)y = 0$

$m(m^2 - 7) - 6$

~~$(m^2 - 7)(m - 6) = 0$~~

~~$m = 6$~~ $\quad m = -1, -2, 3$

~~$m^2 = 7$~~

~~$m = \sqrt{7}$~~

$C.F. = C_1 e^{-x} + C_2 e^{-2x} + C_3 e^{3x}$

$y = C.F. + P.I.$

$y = C_1 e^{-x} + C_2 e^{-2x} + C_3 e^{3x}$