

The following mortality rates have been obtained for a certain type of light bulb.

week	1	2	3	4	5
# percent	10	25	50	80	100

there are 100 bulbs in use and it costs Rs. 1.00 to replace an individual bulb which has worked out.

If all bulbs were replaced simultaneously it would cost 25 paise/bulb. If it is proposed to replace all bulbs at fixed intervals, whether or not they have worked out and to continue replacing worked out bulbs as they fail.

At what interval should all the bulbs be replaced

If P_i is the probability of failure of a new bulb in the highest i th week then,

$$P_1 = \frac{10}{100} = \boxed{0.10}$$

$$P_2 = \frac{25-10}{100} = \boxed{0.15}$$

$$P_3 = \frac{50-25}{100} = \boxed{0.25}$$

$$P_4 = \frac{80-50}{100} = \boxed{0.3}$$

$$P_5 = \frac{100-80}{100} = \frac{20}{100} = \frac{1}{5} = \boxed{0.2}$$

hence, sum of probability —

$$P_1 + P_2 + P_3 + P_4 + P_5 = 1$$

$$0.10 + 0.15 + 0.25 + 0.30 + 0.20 = 1$$

$$1 = 1$$

Therefore a bulb can't survive for more than 5 week.
i.e. A Bulb which has survive for 4 weeks insure
fail in the 5th week.

Assuming that worked out Bulb in any week are
replaced just at the end of that worked.

If N_i be the no. of replacement at the
end of i^{th} week.

while all 1000 Bulbs were new initially then
we have

$$N_0 = \text{Number of Bulbs in the beginning} = 1000$$

$$N_1 = \text{No. of worked out bulbs replaced at the 1st week.}$$
$$= N_0 P_1 = 1000 \times 0.10 = 100$$

$$N_2 = N_0 P_2 + N_1 P_1 = 1000 \times 0.15 + 100 \times 0.10 = 160$$

$$N_3 = N_0 P_3 + N_1 P_2 + N_2 P_1 = 1000 \cdot 0.25 + 100 \cdot 0.15 + 160 \cdot 0.10 = 281$$

$$N_4 = 377$$

$$N_5 = 350$$

$$N_6 = 230$$

$$N_7 = 286$$

$$N_8 = 320$$