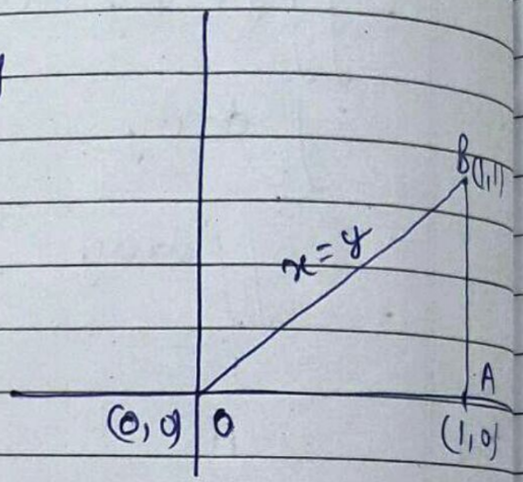
Ques- Using Green Theorem, evaluate $\int_C (x^2 y\, dx + x^2 dy)$ where C is the boundary described counter clockwise of the triangle with vertices (0,0), (1,0) and (1,1).

By Green theorem.
We have

$$\int_C (M dx + N dy) = \iint_S \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dx\, dy$$



Here $M = x^2 y$ and $N = x^2$

$$\frac{\partial N}{\partial x} = 2x, \quad \frac{\partial M}{\partial y} = x^2$$

$$\Rightarrow \int_0^1 \int_0^x (2x - x^2)\, dy\, dx$$

$$= \int_0^1 \left[2xy - x^2 y\right]_0^x dx$$

$$= \int_0^1 (2x^2 - x^3)\, dx$$

$$= \int_0^1 2x^2 dx - \int_0^1 x^3\, dx$$

$$= 2\left[\frac{x^3}{3}\right]_0^1 - \left[\frac{x^4}{4}\right]_0^1$$

$$= \frac{2}{3} - \frac{1}{4}$$

$$= \frac{8-3}{12}$$

$$= \frac{5}{12}$$

Ques- Evaluate $\int_C (y-\sin x)\,dx + \cos x\,dy$ where C is the triangle formed by $y=0$, $x=\frac{\pi}{2}$, $y=\frac{2}{\pi}x$

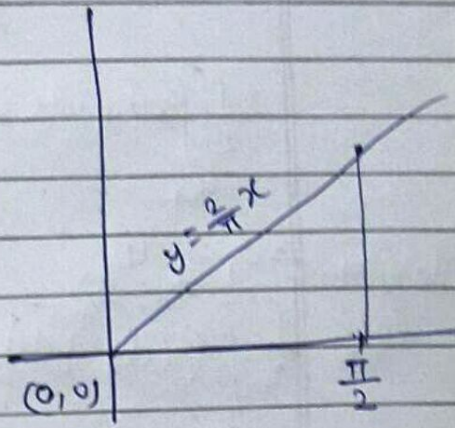
By Green Theorem,

We have

$$\int_C M\,dx + N\,dy = \iint_S \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dx\,dy$$

Here $M = y-\sin x$ and $N = \cos x$

$$\frac{\partial M}{\partial y} = 1, \quad \frac{\partial N}{\partial x} = -\sin x$$



$$\int_C (y-\sin x)\,dx + \cos x\,dy$$

$$= \int_0^{\pi/2}\int_0^{2/\pi\, x} (-\sin x - 1)\,dy\,dx$$

$$= \int_0^{\pi/2} \left[-\sin x\, y - y\right]_0^{\frac{2x}{\pi}} dx$$

$$= \int_0^{\pi/2} \left[-\sin x \frac{2x}{\pi} - \frac{2x}{\pi}\right] dx$$

$$= \cancel{\int_0} -\frac{2}{\pi}\int_0^{\pi/2} \cancel{\sin x}\, x(\sin x + 1)\,dx$$

$$= \frac{2}{\pi} = -\left(\frac{\pi}{4} + \frac{2}{\pi}\right)$$

Ques- Using Green theorem, find the area of the region in the first quadrant bounded by the curves

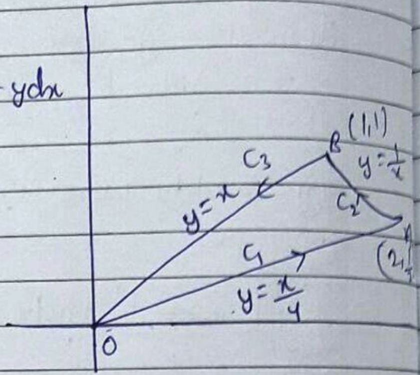
$$y = x, \quad y = \frac{1}{x}, \quad y = \frac{x}{4}$$

Sol^n- By Green Theorem,

Area of the ~~bounded~~ region bounded by given curves

$$A = \frac{1}{2}\int_C x\,dy - y\,dx$$

$$= \frac{1}{2}\left[\int_{C_1} x\,dy - y\,dx + \int_{C_2} x\,dy - y\,dx + \int_{C_3} x\,dy - y\,dx\right] \quad (3)$$



Along C_1, $y = \frac{x}{4}$, $dy = \frac{dx}{4}$ (1) (2)

Eq (1) $\frac{1}{2}\int_0 (x\,dy - y\,dx)$

$$= \frac{1}{2}\int \frac{x\,dx}{4} - \frac{dx\,x}{4}$$

$= 0.$

Along C_3, $y = x$, $dy = dx$

$$\frac{1}{2}\int x\,dy - y\,dx = \frac{1}{2}\int x\,dx - x\,dx$$

$= 0$

Along C_2, $y = \frac{1}{x}$, $dy = \frac{-dx}{x^2}$

$$\frac{1}{2}\int x\,dy - y\,dx = \frac{1}{2}\int x\left(\frac{-dx}{x^2}\right) - \frac{1}{x}dx$$

$$= \frac{1}{2}\int \frac{-dx}{x} - \frac{dx}{x}$$

$$= \frac{2}{2}\int_2^1 -\frac{dx}{x} = -\log[x]_2^1 = -~~2\log 2~~$$

$$= -[0 - \log 2]$$

$= \log 2$

$A = 0 + 0 + \log 2$

$= \log 2$