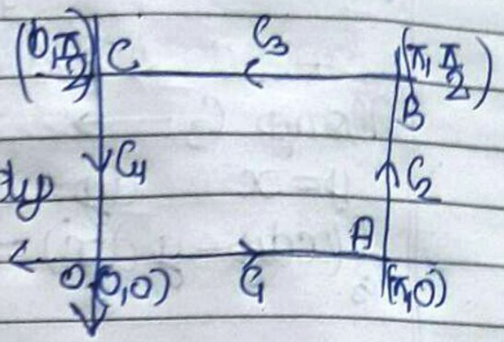
$-(e^{\pi}-1)$
$-(\sin\frac{\pi}{2}-0)$

Q. Evaluate by Green Theorem —
$\int_C (e^{-x}\sin y\,dx + e^{-x}\cos y\,dy)$ where C is the rectangle with vertices $(0,0)$ $(\pi,0)$ $\left(\pi,\frac{\pi}{2}\right)$, $\left(0,\frac{\pi}{2}\right)$ and hence verify green theorem.

By Green Theorem ——



$$\int_C (M\,dx + N\,dy) = \iint_S \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dx\,dy$$

$$\int_C (e^{-x}\sin y\,dx + e^{-x}\cos y\,dy) = \iint_S (-e^{-x}\cos y - e^{-x}\cos y)\,dx\,dy$$

$$= -2\int_0^{\pi/2}\int_0^{\pi} e^{-x}\cos y\,dx\,dy$$

$$= -2\int_0^{\pi/2} \left[-e^{-x}\right]_0^{\pi} \cos y\,dy$$

$$= -2\,(1-e^{-\pi})\left[+\sin y\right]_0^{\pi/2}$$

$$= +2\,(1-e^{-\pi})\,(-1)$$

$$= -2\,(1-e^{-\pi})$$

$$= 2(e^{-\pi}-1) \quad \underline{\text{Ans}}$$

RHS $= 2(e^{-\pi}-1)$

$$LHS = \int_C (e^{-x}\sin y\,dx + e^{-x}\cos y\,dy)$$

$$= \int_{C_1} (e^{-x}\sin y\,dx + e^{-x}\cos y\,dy) + \int_{C_2} (e^{-x}\sin y\,dx + e^{-x}\cos y\,dy)$$
$$+ \int_{C_3} (e^{-x}\sin y\,dx + e^{-x}\cos y\,dy) + \int_{C_4} (e^{-x}\sin y\,dx + e^{-x}\cos y\,dy)$$

Along $C_1 \longrightarrow$

$y=0$, $dy=0$

$$\int_{C_1} (e^{-x}\sin y\,dx + e^{-x}\cos y\,dy) = 0$$

Along $C_2 \longrightarrow$

$x=\pi$, $dx=0$

$$\int_{C_2} (e^{-x}\sin y\,dx + e^{-x}\cos y\,dy) = \int_0^{\pi/2} e^{-\pi}\cos y\,dy$$
$$= e^{-\pi}[\sin y]_0^{\pi/2}$$
$$= e^{-\pi}$$

Along $C_3 \longrightarrow$

$y=\pi/2$, $dy=0$

$$\int_{C_3} (e^{-x}\sin y\,dx + e^{-x}\cos y\,dy) = \int_\pi^0 e^{-x}\,dx$$
$$= -[e^{-x}]_\pi^0$$
$$= (e^{-\pi}-1)$$

Along $C_4 \longrightarrow$ $x=0$ $dx=0$

$$\int_{C_3} (e^{-x}\sin y\,dx + e^{-x}\cos y\,dy) = \int_{\pi/2}^0 \cos y\,dy$$
$$= [\sin y]_{\pi/2}^0$$
$$= -1$$

from eq-①, we get →

$$LHS = 0 + e^{-\pi} + e^{-\pi} - 1 - 1$$
$$= 2(e^{-\pi} - 1)$$

$$LHS = RHS$$

Hence, green theorem is verified.