Conservative Vector Field: If the vector field $\vec{F}$ is the gradient of a scalar value function $f$, then $\vec{F}$ is called a conservative vector field. $\boxed{\vec{F} = \nabla f}$

Eg If $\vec{F} = (2x, 2y) = 2x\hat{i} + 2y\hat{j}$ then show that the vector field $\vec{F}$ is conservative.

Let $f = x^2 + y^2$ then —

$\nabla f = \hat{i}\,2x + \hat{j}\,2y = \vec{F}$

Therefore $\vec{F}$ is conservative

## Fundamental Theorem of line Integral :-

Let $\vec{F} = \nabla f$ be a conservative vector field along a curve C from A to B. Then —

$$\boxed{\int_A^B \vec{F}\cdot d\vec{r} = \int_C \nabla f\cdot d\vec{r} = f(B) - f(A)}$$

**Q.** Let $\vec{F}(x,y,z) = (2xyz + e^{x+z}, x^2z, x^2y + e^{x+z})$ and let C be a path from (1,2,0) to (1,0,5)

Compute → $\int_C \vec{F}\cdot d\vec{r}$

Let $f = x^2yz + e^{x+z}$

Now $\nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right)$

$\nabla f = (2xyz + e^{x+z}, x^2z, x^2y + e^{x+z})$

$\nabla f = \vec{F}$

Here f is the scalar ~~differ~~ potential of $\vec{F}$.

By Fundamental Theorem of line Integral —

$$\int_A^B \vec{F}\cdot d\vec{r} = \int_C \nabla f\cdot d\vec{r} = f(B) - f(A)$$

$$\int_C \nabla f\cdot d\vec{r} = f(B) - f(A)$$
$$= f(1,0,5) - f(1,2,0)$$
$$= e^6 - e \quad \text{Ans}$$

Q. Evaluate $\int_1^2 x^3 dx$ by FTLI.

$\vec{F} = x^3$

$f = \frac{x^4}{4}$

$\vec{F} = \nabla f$

By Fundamental Theorem of Line Integral -

$\int_A^B \vec{F} \cdot d\vec{r} = f(B) - f(A)$

$\int_1^2 x^3 dx = f(2) - f(1)$

$= \frac{2^4}{4} - \frac{1}{4}$

$= \frac{15}{4}$ Ans