Note:- ① Direction in which directional derivative is maximum = grad ϕ = $\nabla\phi$

② Maximum value of the directional derivative = $|\text{grad}\,\phi| = |\nabla\phi|$

③ The directional derivative is maximum along the normal to the surface.

Q. Find the DD of the function $f = x^2 - y^2 + 2z^2$ at the point P(1,2,3) in the direction of the line PQ where Q is the point (5,0,4). In what direction it will be maximum. Find also the magnitude of this maximum.

$DD = \nabla f \cdot \hat{n}$

$\nabla f = 2x\hat{i} - 2y\hat{j} + 4z\hat{k}$

$\vec{PQ} = \vec{OQ} - \vec{OP}$

$\vec{n} = (5\hat{i} + 0\hat{j} + 4\hat{k}) - (\hat{i} + 2\hat{j} + 3\hat{k})$

$\vec{n} = 4\hat{i} - 2\hat{j} + \hat{k}$

$\hat{n} = \frac{1}{\sqrt{21}}(4\hat{i} - 2\hat{j} + \hat{k})$

$DD = \nabla f \cdot \hat{n}$

$DD = \frac{1}{\sqrt{21}}(8x + 4y + 4z)$

At $(1,2,3) \longrightarrow$

$DD = \frac{1}{\sqrt{21}}(8 + 8 + 12)$

$= \frac{28}{\sqrt{21}}$

$DD = \frac{4\sqrt{21}}{3}$ Ans

The DD of f is maximum in the direction of the normal to the given surface i.e. in the direction of $\nabla f = 2\hat{i} - 4\hat{j} + 12\hat{k}$

Maximum value of the directional derivative $= |\nabla f|$

$= \sqrt{4 + 16 + 144}$

$= \sqrt{164}$ Ans

Q. Find the values of constants a, b, c so that the maximum value of the directional derivative of $\phi = axy^2 + byz + cz^2x^3$ at $(1, 2, -1)$ has a magnitude 64 in the direction parallel to z-axis.

$\hat{n} = \hat{k}$

$\phi = axy^2 + byz + cz^2x^3$

$\nabla\phi = (ay^2 + 3cz^2x^2)\hat{i} + (2axy + bz)\hat{j} + (by + 2czx^3)\hat{k}$

At $(1, 2, -1) \longrightarrow$

$\nabla\phi = (4a + 3c)\hat{i} + (4a - b)\hat{j} + (2b - 2c)\hat{k}$

$4a + 3c = 0$ $\qquad$ $2b - 2c > 0$

$4a - b = 0$

$DD = \nabla\phi \cdot \hat{n} = 2(b - c)$

$2(b - c) = 64$

$b - c = 32$

$b + 3c = 0$

$-4c = 32$

$c = -8$

$b = -8 + 32$

$b = 24$

$4a = 24$

$a = 6$