Date ___/___/______

→ Method of Variation of parameter –

steps for finding solutions –

form –

$$\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = R$$

1. Find out the part of C.F.
2. Let them be u and v
3. Consider $y = Au + Bv$ be the complete solution of given diff. eq^n.
4. where, $A = \int \frac{-Rv}{uv_1 - u_1v}\,dx + C_1$

$$B = \int \frac{Ru}{uv_1 - u_1v}\,dx + C_2$$

$$\left(u_1 = \frac{du}{dx}\right)$$

$$\left(v_1 = \frac{dv}{dx}\right)$$

Q. Solve – the following differential equation By the method of variation of parameters –

$$\frac{d^2y}{dx^2} + a^2y = \sec ax$$

$$R = \sec ax$$

$$(D^2 + a^2)y = 0$$

$$D^2 = -a^2$$

$$D = \pm ai$$

Date ___/___/___

$C.P. = (b\cos ax + c\sin ax)$

Part of C.F. of given diff. eq^n –

$u = \cos ax \quad \to$ Let $y = Au + Bv$ (complete sol.)

$v = \sin ax$

$$A = -\int \frac{\sec ax \sin ax}{a\cos^2 ax + a\sin^2 ax}\,dx$$

$$= -\int \frac{\tan ax}{a}\,dx + c_1$$

$$= -\frac{1}{a^2}(\log \sec ax) + c_1 = \frac{1}{a^2}(\log \cos ax) + c_1$$

$$B = \int \frac{\sec ax \cos ax}{a}\,dx$$

$$B = \int \frac{1}{a}\,dx = \frac{x}{a} + c_2$$

complete solution –

$$y = \cos ax\left[\frac{(\log \cos ax)}{a^2} + c_1\right] + \sin ax\left[\frac{x}{a} + c_2\right]$$