solution of ~~first~~ system of first order differentiable eqn. using matrix method :-

Que. Find the general solution of the system of equations

$$y_1' = -2y_1 + y_2$$

$$y_2' = y_1 - 2y_2$$

Sol. Matrix form of the given system is.

$$\begin{bmatrix} y_1' \\ y_2' \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

$y' = Ay$, where, $y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$, $A = \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix}$

put $y = e^{\lambda t} \cdot x$, we obtain eigen values problem,

$$y' = \lambda e^{\lambda t} \cdot x$$

$$\lambda \cancel{e^{\lambda t}} \cdot x = A \cancel{e^{\lambda t}} x$$

$$(\lambda I - A)x = 0$$

The characteristic eqn of ~~A~~

$$\lambda^2 + 4\lambda + 3 = 0$$

$$\lambda^2 + (3+1)\lambda + 3 = 0$$

$$\lambda(\lambda + 3) + 1(\lambda + 3) = 0 \qquad \lambda = -1, \lambda = -3$$

corresponding to $\lambda = -1$

$$x^{(1)} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

corresponding to $\lambda = -3$

$$x^{(2)} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

General solution of the given system is

$$y = A_1 e^{\lambda_1 t} x^{(1)} + B_1 e^{\lambda_2 t} x^{(2)}$$

$$\boxed{y = A_1 e^{-t} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + B_1 e^{-3t} \begin{bmatrix} 1 \\ -1 \end{bmatrix}}$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = y = A_1 e^{-t} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + B_1 e^{-3t} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Component wise solution

$$y_1 = A_1 e^{-t} + B_1 e^{-3t}$$

$$y_2 = A_1 e^{-t} - B_1 e^{-3t}$$