Q - Solve the following differential eqn

$$(D^2 - 4m + 4)y = 8x^2 e^{2x} \sin 2x.$$

Solution:- $m = 2, 2$

$$P.I. = \frac{1}{(D-2)^2} 8x^2 e^{2x} \sin 2x$$

$$= 8 e^{2x} \cdot \frac{1}{D^2} x^2 \sin 2x$$

$$P.I. = 8 e^{2x} \cdot \frac{1}{D} \int x^2 \sin 2x \, dx$$

$$\int x^2 \sin 2x \, dx = -\frac{x^2}{2} \cos 2x + \int \cancel{2}x \cdot \frac{\cos 2x}{\cancel{2}} dx$$

$$= -\frac{x^2}{2} \cos 2x + \frac{x}{2} \sin 2x - \int \frac{\sin 2x}{2} dx$$

$$= -\frac{x^2}{2} \cos 2x + \frac{x}{2} \sin 2x + \frac{1}{2} \frac{\cos 2x}{2}$$

$$= -\frac{x^2}{2} \cos 2x + \frac{x}{2} \sin 2x + \frac{1}{4} \cos 2x$$

$$PI = 8 e^{2x} \frac{1}{D}\left[-\frac{x^2}{2} \cos 2x + \frac{x}{2} \sin 2x + \frac{1}{4} \cos 2x\right]$$

$$= e^{2x}\left[(3 - 2x^2) \sin 2x - 4x \cos 2x\right]$$

Q - Find the complete solⁿ of

$(D^2+a^2)\,y = \sec ax$

Sol :- $m = \pm ai$

$$PI = \frac{1}{D^2+a^2}\sec ax = \frac{1}{2ia}\left[\frac{1}{D-ia} - \frac{1}{D+ia}\right]\sec ax$$

$$= \frac{1}{2ia}\left[e^{iax}\int e^{-iax}\sec ax\,dx - e^{-iax}\int e^{iax}\sec ax\,dx\right]$$

$$= \frac{1}{2ia}\left[e^{iax}\int(\cos ax - i\sin ax)\frac{1}{\cos ax}dx - e^{-iax}\int(\cos ax + i\sin ax)\frac{1}{\cos ax}dx\right]$$

$$= \frac{1}{2ia}\left\{e^{iax}\int(1 - i\tan ax)\,dx - e^{-iax}\int(1+i\tan ax)\,dx\right]$$

$$= \frac{1}{2ia}\left[e^{iax}\left\{x + \frac{i}{a}\log\cos ax\right\} - e^{-iax}\left\{x - \frac{i}{a}\log\cos ax\right\}\right]$$

$$= \frac{1}{2ia}\left[x\left(e^{iax} - e^{-iax}\right) + i\left(\frac{\log\cos ax}{a}\right)\left(e^{iax} + e^{-iax}\right)\right]$$

$$= \frac{1}{2ia}\left[2ix\sin ax + \frac{2i}{a}\log\cos ax \cdot \cos ax\right]$$

$$= \frac{1}{a}\left[x\sin ax + \frac{1}{a}\cos ax \cdot \log\cos ax\right] //$$