divide a +ve quantity c into n parts in such a way as to maximize their products.
Now Mathematically this problem formulated as

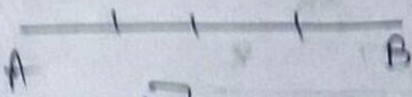
or Max $z = y_1 . y_2 . \cdots . y_n$

s.t.

$y_1 + y_2 + \cdots + y_n = c$

$y_j \geq 0$, for $j = 1, 2, \cdots, n$

Sol: Let $f_n(c) \rightarrow$ denote the maximum attainable product, when c divided into n parts.




$$f_n(c) = \max_{0 < x \leq c} \left[ x f_{n-1}(c-x) \right] \quad n = 2, 3, 4, \cdots$$

for n=1

$$f_1(c) = \max_{y_1 = c} \{ y_1 \} = c \text{ (trivially)}$$

for n=2

$$f_2(c) = \max_{0 < x \leq c} \left[ x f_1(c-x) \right]$$

$$= \max_{0 < x \leq c} \left[ x(c-x) \right]$$

$$\boxed{f_2(c) = \left(\frac{c}{2}\right)^2}$$

$$\frac{d}{dx} x(c-x) = 0 \Rightarrow x = c/2$$

max at $x = c/2$

∴ the optimal policy is $\left(\frac{c}{2}, \frac{c}{2}\right)$ & $f_2(c) = \left(\frac{c}{2}\right)^2$

n=3,

$$f_3(c) = \max_{0 < x \leq c} \left[ x f_2(c-x) \right]$$

$$= \max_{0 < x \leq c} \left[ x \left(\frac{c-x}{2}\right)^2 \right]$$

$$\boxed{f_3(c) = \left(\frac{c}{3}\right)^3}$$

optimal policy is $\left(\frac{c}{3}, \frac{c}{3}, \frac{c}{3}\right)$ & $f_3(c) = \left(\frac{c}{3}\right)^3$

⋮

$$f_m(c) = \left(\frac{c}{m}\right)^m$$

$$f_{m+1}(c) = \max_{0 \le x \le c} \left[ x \, f_m(c-x) \right]$$

$$= \max_{0 \le x \le c} \left[ x \left(\frac{c-x}{m}\right)^m \right]$$

$$\boxed{f_n(c) = \left(\frac{c}{n}\right)^n}$$

$$\left(\frac{c}{n}, \frac{c}{n}, \dots, \frac{c}{n}\right)$$

$$\frac{d}{dx}\left[ x \left(\frac{c-x}{m}\right)^m \right] = 0$$

$$x = \frac{c}{m+1}$$

$$f_{m+1}(c) = \left(\frac{c}{m+1}\right)^{m+1}$$

Hence by mathematical induction the optimal policy is $\left(\frac{c}{n}, \frac{c}{n}, \dots, \frac{c}{n}\right)$ & $f_n(c) = \left(\frac{c}{n}\right)^n$ proved