→ Method of change of independent variable -

form - $\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = R$

steps - (1.) choose z such that

$$\left(\frac{dz}{dx}\right)^2 = Q$$

Here, Q is taken in such a way that it remains the whole square of a function without surd and its negative sign is ignored

(2.) Find the value of -

$$P_1 = \frac{\frac{d^2z}{dx^2} + P\frac{dz}{dx}}{\left(\frac{dz}{dx}\right)^2}$$

$$Q_1 = \frac{Q}{\left(\frac{dz}{dx}\right)^2}$$

$$R_1 = \frac{R}{\left(\frac{dz}{dx}\right)^2}$$

(3.) Putting the above values of P_1, Q_1 and R_1 in - $\frac{d^2y}{dz^2} + P_1\frac{dy}{dz} + Q_1y = R_1$

which we solve for y in terms of z.

Q. Solve the following differential eq^n by changing the independent variable-

$$\frac{d^2y}{dx^2} - \frac{1}{x}\frac{dy}{dx} + 4x^2 y = x^4$$

$\left(\begin{array}{l}\sqrt{2}x^2 \\ \rightarrow x^2\end{array}\right)$

Sol. $P = \frac{-1}{x}$; $Q = 4x^2$; $R = x^4$

$$\left(\frac{dz}{dx}\right)^2 = 4x^2$$

$$\frac{dz}{dx} = 2x$$

$$z = \frac{2x^2}{2} = x^2$$

$$P_1 = \frac{2 + \frac{-1}{x}\cdot 2x}{4x^2} = 0$$

$$Q_1 = \frac{4x^2}{4x^2} = 1$$

$$R_1 = \frac{x^2}{4}$$

$$\frac{d^2y}{dz^2} + 0\cdot\frac{dy}{dz} + y = \frac{x^2}{4}$$

$$\frac{d^2y}{dz^2} + y = \frac{x^2}{4}$$

$$\frac{d^2y}{dz^2} + y = \frac{z}{4}$$

$$(4D^2 + 4)y = z$$

$$4D^2 + 4 = 0$$

$$D^2 = -1$$

$$D = \pm i \Rightarrow m = \pm i$$

$\Rightarrow C.F. = c_1 \cos z + c_2 \sin z$

$\Rightarrow P.I. = \frac{1}{(D^2+1)} \cdot \frac{z}{4}$

$= \frac{1}{4}\left[1 + D^2\right]^{-1} \cdot z$

$= \frac{1}{4}\left(1 - D^2\right) z$

Date ___/___/____

P.I. $= \frac{1}{4} \cdot z = \frac{z}{4}$

$y = C_1 \cos x + C_2 S_1$

$y = C_1 \cos z + C_2 \sin z + \frac{z}{4}$

$y = C_1 \cos x^2 + C_2 \sin x^2 + \frac{x^2}{4}$