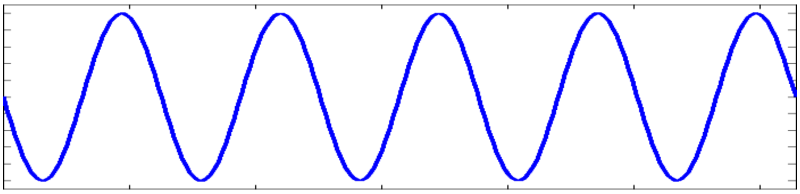
Mathematical Analysis of Interference of light

Let there are two light waves of same frequency and different amplitudes a_1 and a_2. Then the equations of waves (resultant displacement due to waves) at any time 't', are given as:

$$y_1 = a_1 \sin \omega t, \quad \ldots\ldots.(1)$$

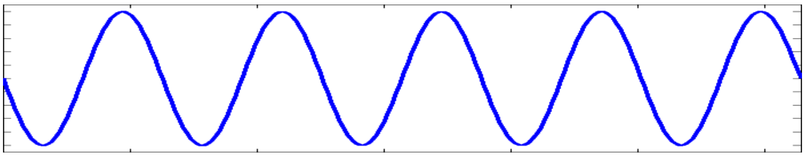
$$y_2 = a_2 \sin(\omega t + \delta), \quad \ldots\ldots.(2)$$

where 'δ' is the phase difference between the two waves.



y_1

+



y_2

According to the principle of superposition, the resultant displacement

$$y = y_1 + y_2 = a_1 \sin \omega t + a_2 \sin(\omega t + \delta)$$

$$y = a_1 \sin \omega t + a_2 \sin \omega t \cos \delta + a_2 \cos \omega t \sin \delta$$

or $$= (a_1 + a_2 \cos \delta) \sin \omega t + a_2 \sin \delta \cos \omega t \quad \ldots\ldots(3)$$

Let $$a_1 + a_2 \cos \delta = A \cos \phi \quad \ldots\ldots(4)$$

and $$a_2 \sin \delta = A \sin \phi \quad \ldots\ldots.(5)$$

where A and ϕ are new unknown constants

On using (4) and (5) in equation (3) we get –

$$y = A \cos \phi \sin \omega t + A \sin \phi \cos \omega t$$

$$y = A \sin(\omega t + \phi) \quad \ldots\ldots.(6)$$

hence resultant vibration at P is simple harmonic of amplitude A and phase ϕ.

Squaring and addition eq^{n}s (4) and (5).

$$(a_1 + a_2 \cos \delta)^2 + (a_2 \sin \delta)^2 = A^2$$

or $$A^2 = a_1^2 + a_2^2 \cos^2 \delta + 2a_1 a_2 \cos \delta + a_2^2 \sin^2 \delta$$

or $$A^2 = a_1^2 + a_2^2 + 2a_1a_2\cos\delta$$

$$A = \sqrt{a_1^2 + a_2^2 + 2a_1a_2\cos\delta}$$

which is the resultant distribution or amplitude.

The resultant intensity is given by –

We know Intensity α (Amplitude)^2

$$I = a_1^2 + a_2^2 + 2a_1a_2\cos\delta \qquad \text{.......(7)}$$

or $$I = a_1^2 + a_2^2 + 2a_1a_2(2\cos^2\delta/2 - 1)$$

or $$= a_1^2 + a_2^2 + 2a_1a_2 + 4a_1a_2\cos^2\delta/2$$

or $$I = (a_1 - a_2)^2 + 4a_1a_2\cos^2\delta/2 \qquad \text{.......(8)}$$

$I \neq a_1^2 + a_2^2$, this shows if a_1 and a_2 are constant, the intensity I will vary from point to point in the region of interference of two waves, according to the variation of $\cos^2\delta/2$. The intensity variation is of the cosine square ($\cos\delta$) form.

Phase difference ϕ, ;

by eq^n (5) and eq^{ns} (4), $$\boxed{\tan\phi = \frac{a_2\sin\delta}{a_1 + a_2\cos\delta}}$$