Subject Name: Mathematic II

Topic: Partial Order Relations and Lattices Part -1

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**Partial order Relation**

Let A be non-empty set and R be a relation defined on A. Then R is said to be Partial order Relation if it is

(i) Reflexive i.e. ${}_aR_a \; \forall \; a \in A$

(ii)Anti- Symmetric i.e. ${}_aR_b, {}_bR_a \Rightarrow a = b \;\; \forall \; a, b \in A$

(iii) Transitive i.e. ${}_aR_b \; and \; {}_bR_c \; \Rightarrow \; {}_aR_c \; \forall a, b, c \in A$

Partial Order Set (Po-set) - A set A together with a partial order relation R in it is called Partially ordered set (Po-set) and is denoted by (P, R)

**Q1.** Let N be the set of positive Integers prove that the relation $\leq$ where $\leq$ has its usual meaning a partial order relation on N.

Solution – Given – N be a positive Integers

Aim

Show that $\leq$ is a Partial order relation

(i) <u>Reflexive relation</u> $\forall \; a \in N$

$\Rightarrow a \leq a$

$\Rightarrow a$ is less than or equal to $a$ itself

$\leq$ is reflexive

(ii) <u>Anti-Symmetric relation</u> Let $\forall a, b \in N$ and

$a \leq b, b \leq a \;\; then \;\; a = b$

Hence the relation $\leq$ is Anti-Symmetric

(iii) Transitive Relation Let $a, b, c \in N$ and

$a \le b, b \le c$ *then* $a \le c$

$\le$ is Transitive

Hence $\le$ is a Partial order relation

**Q2.** Prove that the relation "a divides b", if there exist a positive integer and is obtained by a | b, on the set of all positive integers N is a partial order relation.

Solution (i) <u>Reflexive relation</u> $\forall \ a \in N$

$$\Rightarrow a \mid a$$

Hence the relation '|' is reflexive

(ii) <u>Anti-Symmetric relation</u> Let $a, b \in N$ and

$a \mid b, b \mid a \Rightarrow$ there exist an integer $c_1, c_2$ such that $ac_1 = b, bc_2 = a$

$$c_1 c_2 = \frac{b}{a} \frac{a}{b}$$

$$\Rightarrow c_1 c_2 = 1$$
$$\Rightarrow c_1 = 1 = c_2$$
$$\Rightarrow a = b$$

Hence the relation '|' is Anti-Symmetric

(iii) Transitive Relation Let $a, b, c \in N$ and $a \mid b, b \mid c \Rightarrow \exists$ positive integer $d_1, d_2$ such that $ad_1 = b, \ bd_2 = c$

$$a d_1 d_2 = c$$

$\Rightarrow \exists$ an integer $d$ such that $ad = c,$

$$\Rightarrow a \mid c$$

Hence the relation '|' is Transitive

Thus, the relation ‘|’ is a Partial order relation

## Referential Books:

1. S.K. Sarkar, “Discrete Maths”; S. Chand & Co.,2000.
2. H.K. Dass, “Advanced Engineering Mathematics”, S. Chand & Co., 9th RevisedEd.
3. “Discrete Mathematics”, Schaum’s Outlines.