Case III. When the thickness of plate is such that $\delta = \pi, 3\pi, 5\pi, \cdots (2n+1)\pi$, then $\sin\delta = 0$ and $\cos\delta = -1$, hence equ^n (6) reduces to

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{2xy}{ab} = 0$$

or $$\left(\frac{x}{a} + \frac{y}{b}\right)^2 = 0 \text{ or } y = -\frac{b}{a}x \quad \cdots \quad (9)$$

This is again the equ^n of a straight line passing through the origin P, Thus the emergent light is again plane polarised having vibrations making an angle $2\tan^{-1}(b/a)$ with that of incident light as shown in fig (5).

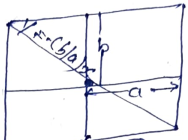
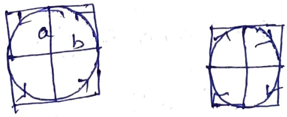


fig (5)

Case (IV) When $a = b$, & $\delta = (2n+1)\pi/2$, $n = 0, 1, 2 \cdots$ then ~~then~~ $\sin\delta = 1$, $\cos\delta = -1$, hence equ^n (6) reduces to $x^2 + y^2 = a^2$

This is the equ^n of a circle of radius a, ∴ Thus the emergent light is circularly polarised as shown in fig (6)

~~reduces to $x^2+y^2=a^2$~~



$\delta = \pi/2, 5\pi/2$ $\qquad$ $\delta = 3\pi/2, 7\pi/2$

fig (6)

⇒ When $\delta = (2n+1)\pi/2$, where $n = 1, 2, 3 \cdots$ the emergent light is elliptically polarised and its axes (major & minor) coincide with the axes of the component linear vibration.

⇒ When $\delta = (2n+1)\pi/2$ & $\theta = 45^\circ$ (that is $a = b$), the emergent light is circularly polarised. Thus the plane polarised and circularly polarised light ~~is~~ are the special cases of elliptically polarised light.

This is the equ^n of an oblique ellipse. Thus, in general the emergent light is elliptically polarised. The exact nature of the resultant motion and the light emerging from calcite depends upon the value of δ

Special Cases :-

Case I. When the thickness of the plate is such that $\delta = 0, 2\pi, 4\pi, \cdots 2n\pi$, then $\sin\delta = 0$ and $\cos\delta = 1$, then equ^n (6) reduces to

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab} = 0 \qquad \delta = \text{phase difference}$$

$$\text{or } \left(\frac{x}{a} - \frac{y}{b}\right)^2 = 0 \text{ or } \pm y = \pm (b/a)x \quad \text{—— (7)}$$

3

Thus the motion is described by a straight line, both x & y have the same sign. Thus when two mutually perpendicular plane polarised waves are in phase then the emergent light is plane polarised with vibration in the same plane as in the incident light as shown in fig (3)

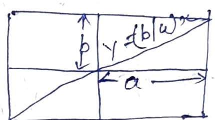


fig. (3)

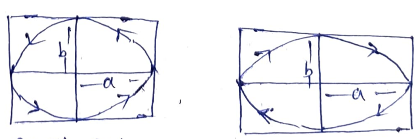
Case II When the thickness of plate is such that $\delta = \pi/2, 3\pi/2 \cdots (2n+1)\pi/2$

where $n = 0, 1, 2 \cdots\cdots$

then $\sin\delta = 1$ and $\cos\delta = 0$, hence equ^n (6) reduce to

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \text{— (8)}$$

This is the equ^n of a simple ellipse, thus when two mutually perpendicular plane polarised waves are in



$\delta = \pi/2, 5\pi/2 \cdots$ $\qquad$ $\delta = 3\pi/2, 7\pi/2$

fig (4)

$\pi/2$ or ~~(2n+1)~~ $(2n+1)\pi/2$ phase then the emergent light is elliptically polarised $(a \neq b)$, the plane of the ellipse being normal to the direction of propagation as shown in fig (4)

~~to the optic axis of amplitude~~ a phase difference δ is introduced between E-wave along PE and O wave along PO in traversing a distance of the plate. In calcite $v_e > v_o$, therefore E-wave leads the O-wave. The displacements of E and O waves along and perpendicular to the optic axis may be expressed by the equations

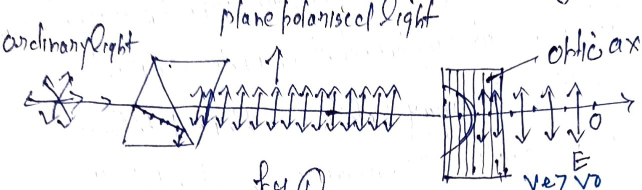


plate calcite crystal

fig ①

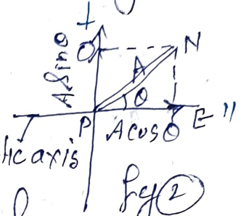


fig ②

$$x = A\cos\theta \sin(\omega t + \delta) \quad \text{—— (1)}$$

$$y = A\sin\theta \sin\omega t \quad \text{—— (2)}$$

Substituting $A\cos\theta = a$ and

and $A\sin\theta = b$ in equ^n (1) & (2), we get

$$\therefore \quad x = a\sin(\omega t + \delta) \quad \text{—— (3)}$$

and

$$y = b\sin\omega t \quad \text{—— (4)}$$

From equ^n (3), we get

$$\frac{x}{a} = \sin\omega t\cos\delta + \cos\omega t \sin\delta \quad \text{—— (5)}$$

From equ^n (4), we have $\frac{y}{b} = \sin\omega t$ and $\sqrt{1-\left(\frac{y}{b}\right)^2} = \cos\omega t$

Substituting these values of $\sin\omega t$ and $\cos\omega t$ in equ^n (5), we get

$$\frac{x}{a} = \frac{y}{b}\cos\delta + \sqrt{\left(1-\frac{y^2}{b^2}\right)}\cdot\sin\delta \text{ or } \frac{x}{a} - \frac{y}{b}\cos\delta = \sqrt{\left(1-\frac{y^2}{b^2}\right)}\cdot\sin\delta$$

Squaring both side, we get

$$\left(\frac{x}{a} - \frac{y}{b}\cos\delta\right)^2 = \left(1 - \frac{y^2}{b^2}\right)\sin^2\delta$$

Solving further, we get $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab}\cos\delta = \sin^2\delta$ —— (6)

Plane, Elliptically and Circularly Polarised Light

Plane polarised light represents the simple type of polarisation for such light the light vector vibrates simple harmonically along a fixed straight line perpendicular to the direction of propagation. When two plane polarised beam of monochromatic light are superimposed, then under suitable conditions the resultant light vector may rotate in a plane ⊥ to the direction of propagation and if its magnitude remains Const while the orientation varies regularly, then the tip of the vector traces a circle and the resultant light is said to be circularly polarised. If, however the magnitude of light vector varies periodically between its maximum and minimum values, the tip of the vector traces an ellipse and the resultant light is said to be elliptically polarised.

Superposition of two plane polarised waves having perpendicular vibrations:

Let a beam of plane polarised light coming from a polariser be incident normally on a thin plate of calcite crystal cut with faces parallel to the optic axis and oriented in such a way that the vibrations in plane polarised light make an angle θ with the optic axis as shown in fig (1) then the amplitude A of the vibrations in the incident plane polarised light be resolved into two components (E & O components) by doubly refracting calcite plate as shown in fig (2) E-wave having vibrations parallel to the optic axis of amplitude $A\cos\theta$ and O-wave having vibrations perpendicular to the optic axis of amplitude $A\sin\theta$. As both the vibrations travel with different velocities in same direction, ~~to the optic axis of amplitude $A\sin\theta$.~~