COUPLED PENDULUM

Object :

To study the angular motion of a coupled pendulum and to find out the beat phenomenon.

Apparatus :

Coupled pendulum set up, stop watch, scale.

Coupled Pendulum Apparatus :

The apparatus consists of two pendulums (S.S. strips) having mass m each at one end. These are connected with a spring having force constant k. The position of the spring can be va[illegible]. Two scales are attached to monitor the given amplitude of these pendulums.

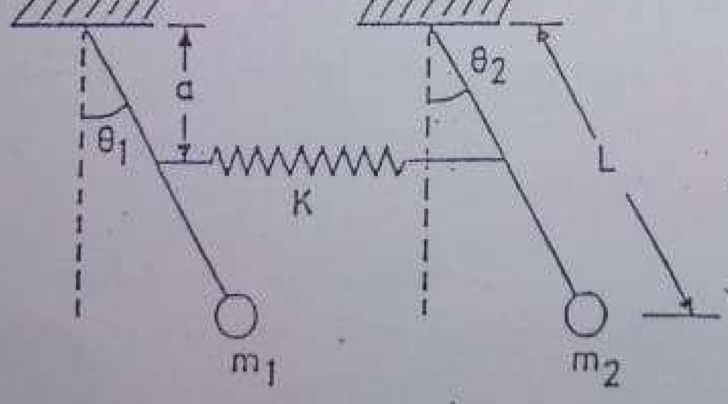

Fig. 1

Theory :

Part I :

$\alpha = \frac{L}{3}, \frac{L}{4}, \frac{L}{5}, L$ etc

For small oscillations

$$ML^2 \frac{d^2\theta_1}{dt^2} = - Mg\, L\, \theta_1 - k\, a^2\, (\theta_1 - \theta_2)$$

and.

$$ML^2 \frac{d^2\theta_2}{dt^2} = - Mg\, L\, \theta_2 + k\, a^2\, (\theta_1 - \theta_2) .$$

Assuming normal mode solutions

$$\theta_1 = A_1 \cos \omega t \qquad (1)$$

and

$$\theta_2 = A_2 \cos \omega t \qquad (2)$$

Natural frequencies are found to be

$$\omega_1 = \sqrt{\frac{g}{L}} \qquad (3)$$

$$\omega_2 = \sqrt{\frac{g}{L} + \left(\frac{2k}{M} \right) \frac{a^2}{L^2}} \qquad (4)$$

where ω_1 and ω_2 are the two natural frequencies of oscillations for first (same phase) and second mode (opposite phase).

For $a = L/3$, measure experimentally the two frequencies ω_1. $\theta_1 = \theta_2 = \theta$ (small) and ω_2, $\theta_1 \neq \theta_2$ (small) for first and second mode by measuring the average time period of free oscillations by stop watch. Do this part for $a = L$ also.

Part II :

Damping factor, ζ will be given by the equation

$$\zeta = \left[\frac{1}{2\pi n} \right] \left[\ln \frac{\theta_0}{\theta_n} \right] \qquad (5)$$

Damping factor $= \left(\frac{1}{2\pi n}\right)\left(\ln \frac{\theta_0}{\theta_n}\right)$

$\theta_0 \rightarrow$ starting amplitude.

$\theta_n \rightarrow$ amplitude after n cycle where $\theta_o \rightarrow$ starting amplitude

$\theta_n \rightarrow$ amplitude after n cycles.

Measure θ_o and θ_n with the help of circular scales are attached in the apparatus and calculate damping factor using relation (5). Repeat experiment three or four times and take the mean. Do this part for $a = L$ also.

Part III :

Beat phenomenon can be observed by starting free oscillations with the following initial conditions,

$$\theta_1 = 0, \quad \theta_2 = \theta \text{ (small)}$$

and

$$\theta_1 = 0, \quad \theta_2 = 0$$

Find out the beat frequency (ω_b) from the average time period of beating with the help of stop watch. Compare this value of ω_b with the value obtained from the first part, i.e.,

$$\omega_b = (\omega_1 - \omega_2) X 2$$

From Equations (3) and (4), we get

$$\left(\omega_2^2 - \omega_1^2\right) = \left(\frac{2k}{M}\right) \cdot \frac{a^2}{L^2} \qquad (6)$$

Plot $\left(\omega_2^2 - \omega_1^2\right)$ vs. $\left(\frac{a^2}{L^2}\right)$ in a graph paper. Slope of this curve will give the value of spring force constant (k).