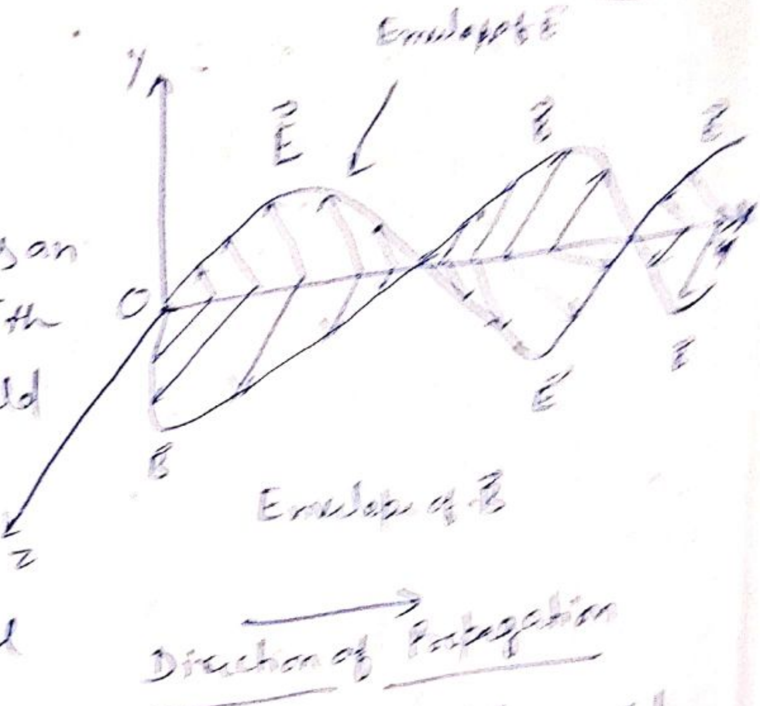
Unit : 3

Electromagnetic wave

according to Maxwell IIIrd equation a changing magnetic field produces an electric field, while according to IV th equation an changing electric field produced magnetic field.

It means change in either field produces the other field. Maxwell worked out from his equations that variation in electric and magnetic fields would lead to a wave consisting of oscillating electric field $\vec{E}$ and magnetic field $\vec{B}$ perpendicular to each other and also perpendi-cular to the direction of propagation of the wave. Such waves which can actually propagate in space even without any material medium are called Electromagnetic waves.



Maxwell predicted also that electromagnetic waves would travel in free space with the speed of light. He thus concluded that light itself is an electromagnetic wave which is transverse in nature.

Other examples of electromagnetic waves are radio-waves, microwaves, infrared rays, ultraviolet rays, X-rays and γ-rays.

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Unit - 03

Electromagnetic wave equation -

Let us Consider a uniform linear medium having permitivity (ϵ) permeability (μ) and Conductivity (σ), but not any charge or current other than that determined by Ohm's law -

Then $\boxed{D = \epsilon E}$ $\boxed{B = \mu H}$ $\boxed{J = \sigma E}$ and $\boxed{\rho = 0}$ ... (1)

and Maxwell's equation,

$$\boxed{\text{div } D = 0, \quad \text{div } B = 0, \quad \text{Curl } E = -\frac{\partial B}{\partial t}, \quad \text{Curl } H = J + \frac{\partial D}{\partial t}} \quad -(2)$$

In this Case, takes the form

$$\text{div } \vec{E} = 0 \quad (3) \qquad \text{div } \vec{H} = 0 \quad (4) \qquad \text{Curl } \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t} \quad (5) \qquad \text{Curl } \vec{H} = \sigma \vec{E} + \epsilon \frac{\partial \vec{E}}{\partial t} \quad (6)$$

Taking curl of (5) $\text{Curl Curl } \vec{E} = -\mu \frac{\partial (\text{Curl } \vec{H})}{\partial t}$ —

Putting the value of Curl $\vec{H}$ from eq (6) then we get.

$$\text{Curl Curl } \vec{E} = -\mu \frac{\partial}{\partial t}\left[\sigma \vec{E} + \epsilon \frac{\partial \vec{E}}{\partial t}\right] = -\sigma\mu \frac{\partial \vec{E}}{\partial t} - \epsilon\mu \frac{\partial^2 E}{\partial t^2} \quad (7)$$

Similarly taking the Curl of eq (6) $\text{Curl Curl } \vec{H} = \text{Curl}\left[\sigma E + \epsilon \frac{\partial \vec{E}}{\partial t}\right]$

$$\text{Curl Curl } \vec{H} = -\sigma\mu \frac{\partial \vec{H}}{\partial t} - \epsilon\mu \frac{\partial^2 H}{\partial t^2} \quad — (8)$$

Now using Vector identity.

$$\boxed{\text{Curl Curl } \vec{A} = \text{grad div } \vec{A} - \nabla^2 A}$$

and keeping view (3) and (4) ($\text{div } \vec{E} = 0$ and $\text{div } \vec{H} = 0$) then eq (7) and (8) will be.

$$\boxed{\begin{aligned} &\nabla^2 E - \sigma\mu \frac{\partial \vec{E}}{\partial t} - \epsilon\mu \frac{\partial^2 \vec{E}}{\partial t^2} \quad — (9) \\ &\nabla^2 H - \sigma\mu \frac{\partial \vec{H}}{\partial t} - \epsilon\mu \frac{\partial^2 H}{\partial t^2} \quad — (10) \end{aligned}}$$

Equation (9) and (10) represent, wave equation which govern the E.M. field in a homogeneous, linear medium, in which the charge density is zero.

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Plane electromagnetic wave in Vaccum -

For free space $\rho = 0$, $\sigma = 0$, $\mu = \mu_0$ and $\epsilon = \epsilon_0$ — (1)

Then maxwell equation will be

$$\text{div } \vec{E} = 0 \quad \ldots (a) \qquad \text{div } \vec{H} = 0 \quad \ldots (b)$$

$$\text{Curl } E = -\mu_0 \frac{\partial \vec{H}}{\partial t} \quad \ldots (c) \qquad \text{Curl } \vec{H} = \frac{\epsilon_0 \partial \vec{E}}{\partial t} \quad \ldots (d) \qquad (2)$$

Taking Curl of 2(c)

$$\text{Curl Curl } \vec{E} = -\mu_0 \frac{\partial}{\partial t}(\text{Curl } \vec{H})$$ and use 2d in this weight

$$\vec{E} = -\mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} \quad \ldots (3)$$

Similarly taking curl of 2(d) and use 2c in it then

$$\text{Curl Curl } \vec{H} = -\epsilon_0 \frac{\partial}{\partial t}(\text{Curl } \vec{E}) = -\epsilon_0 \frac{\partial}{\partial t}\left(\mu_0 \frac{\partial H}{\partial t}\right) = -\mu_0 \epsilon_0 \frac{\partial^2 H}{\partial t^2} \quad - (4)$$

Now using vector Identity -

$$\boxed{\text{Curl Curl } \vec{A} = \text{grad div } \vec{A} - \nabla^2 \vec{A}}$$

And keeping in view 2a and 2b [$\text{div } \vec{E} = 0$ and $\text{div } \vec{H} = 0$]

Then (3) and (4) will be

$$\nabla^2 \vec{E} - \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} = 0 \quad (5)$$

$$\nabla^2 \vec{H} - \mu_0 \epsilon_0 \frac{\partial^2 \vec{H}}{\partial t^2} = 0 \quad (6)$$

Equation (5) and (6) represent wave equation governing Electromagnetic field $\vec{E}$ and $\vec{H}$ in free space. Eq (5) and (6) are similar scalar wave equation

$$\nabla^2 u - \mu_0 \epsilon_0 \frac{\partial^2 u}{\partial t} = 0 \quad - (7)$$

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Here u is scalar and can stand for one of the component of $\vec{E}$ and $\vec{H}$. eq. (7) resemble with the general wave equation

$$\nabla^2 u = \frac{1}{v^2}\frac{\partial^2 u}{\partial t^2} \quad -(8)$$

Here v is velocity of wave. Comparing eq. (7) and (8) we see field vector E and H are propagated in free space as waves at a speed equal to -

$$V = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

~~Which is velocity of light (c)~~

$$\frac{\mu_0}{4\pi} = 10^{-7}$$

$$\frac{\mu_0}{4\pi} = 10^{-7} \qquad 4\pi\epsilon_0 = \frac{1}{9\times10^{9}}$$

$$V = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = \sqrt{\frac{4\pi \quad 9\times10^{9}}{10^{-7}\times 4\pi}} = 3\times10^{8}\ m/sec$$

which is velocity of light -

Therefore, it is reasonable to write c the speed of light

The speed of light in place of $\frac{1}{\mu_0\epsilon_0}$ so eq 5, 6, 7

$$\left.\begin{aligned} \nabla^2\vec{E} - \frac{1}{c^2}\frac{\partial^2\vec{E}}{\partial t^2} &= 0 \\ \nabla^2\vec{B} - \frac{1}{c^2}\frac{\partial^2\vec{H}}{\partial t^2} &= 0 \\ \nabla^2 u - \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} &= 0 \end{aligned}\right| \quad - (9)$$