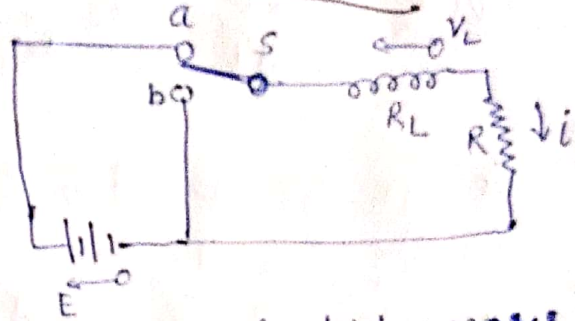
- Growth of Current in an L-R (Inductance-Resistance) Circuit

Circuit ⇒ let us consider a circuit containing a resistance less coil of self-inductance L and a non inductive resistance R connected to battery of constant emf E through a two way switch S. When the circuit is closed (switch S to a) a self induced emf is set up in the coil which opposes the growth of current in the circuit. Hence the current does not reach its final steady value E/R instantly, but grows at a rate depending upon the values of L and R in the circuit.



During the variable state, when the current is growing, let i be the current and di/dt the rate of growth of current at any instant t. Then the instantaneous P.d. across the resistance R is

$$V_R = iR$$

and P.d. across the inductance L is, $V_L = -L\frac{di}{dt}$

Since V_L is opposite to E, the net P.d. that appears across the resistance is $E - L\left(\frac{di}{dt}\right)$. This by ohm's law, must equal to iR. Hence

$$E - L\frac{di}{dt} = iR \quad \text{or} \quad E - iR = L\frac{di}{dt}$$

or $dt = \frac{L\,di}{E - iR}$, Integrating, we get

$$t = -\frac{L}{R}\log_e(E - iR) + C$$

Here C is integration constant. Now at t = 0, i = 0 so

$$C = \frac{L}{R}\log_e E$$

then $t = -\frac{L}{R}\log_e(E - iR) + \frac{L}{R}\log_e E$

or $-\frac{R}{L}t = \log_e(E - iR) - \log_e E = \log\left(\frac{E - iR}{E}\right)$

taking anti log

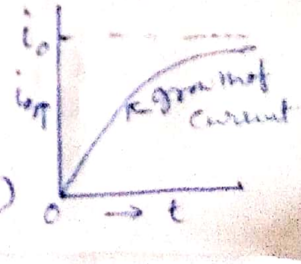
$$e^{-(R/L)t} = \frac{E - iR}{E} = 1 - \frac{iR}{E}$$

$$\frac{iR}{E} = 1 - e^{-(R/L)t}$$

or $i = \frac{E}{R}\left[1 - e^{-(R/L)t}\right]$

But $\frac{E}{R} = i_0$ so final steady value of current

$$i = i_0\left(1 - e^{-(R/L)t}\right) \quad \text{--- (1)}$$



The rate of growth of the Current is obtained by differentiating eq (1),

$$\frac{di}{dt} = i_0 \frac{R}{L} e^{-(R/L)t} \Rightarrow i_0 \frac{R}{L}\left(1-\frac{i}{i_0}\right) \quad \text{from eq (1)}$$

$$= \frac{R}{L}(i_0 - i).$$

So greater the ratio $\frac{R}{L}$ or smaller ratio $\frac{L}{R}$, the more rapidly does the currents approach its maximum value. The ratio $\frac{L}{R}$ is called the 'Inductive time Constant' (τ_L) of the Circuit and expressed in second

If L and R in henry and ohm respectively. Putting $t = \tau_L = \frac{L}{R}$ in eq (1) we get

$$i = i_0(1-e^{-1}) = i_0 \frac{e-1}{e} = i_0 \frac{2.718-1}{2.718} = 0.632\, i_0 \quad (\because e = 2.718)$$

So the Inductive time Constant of an L-R Circuit is the time in which the Current grows from zero to 0.632 of its steady value.

Decay of Current in L-R Circuit.

When the switch S is thrown over to b. the battery is cut off, the L-R Circuit is again closed and the Current in the circuit decays. The self induced emf in the Coil now opposes the decay of the Current, since now E = 0, then equation for decay is

$$-L\frac{di}{dt} = iR \quad \text{or} \quad dt = -\frac{L}{R}\frac{di}{i} \quad (\text{Integrating we get})$$

$$\Rightarrow t = -\frac{L}{R}\log_e i + C \quad (\text{Here C is Integration Constant})$$

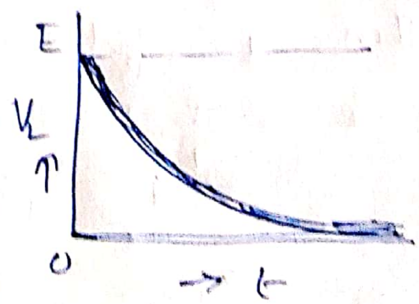
Now at $t = 0$, $i = i_0$ So

$$C = \frac{L}{R}\log_e i_0 . \text{ Then}$$

$$t = -\frac{L}{R}\log_e i + \frac{L}{R}\log_e i_0$$

$$-\frac{R}{L}t = \log_e i - \log_e i_0 = \log_e \frac{i}{i_0}$$

$$e^{-(R/L)t} = i/i_0 \quad \text{or} \quad \boxed{i = i_0 e^{-(R/L)t}} \quad \text{—(2)}$$

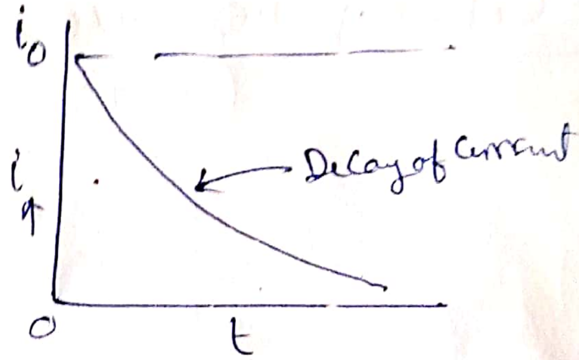


Next to Page-07

Unit - 03

Decay of Current in L.R Circuit

eq (2) is the Helmholtz equation for the decay of current in an L-R. Circuit. It shows that current in L-R. circuit decay exponentially, reaching zero at $t \to \infty$ (asymptotically).



The rate of decay of current is obtained by differentiating eq (2) that is

$$\frac{di}{dt} = -i_0 \frac{R}{L} e^{-(R/L)t} = -i_0 \frac{R}{L} \frac{i}{i_0} \quad \text{(from eq 2)}$$

$$= -\frac{R}{L} i.$$

So it is clear that greater the ratio $\frac{R}{L}$, or smaller the inductive time constant $\frac{L}{R}$, the more rapidly does the current decay. Putting $t = \tau_L = \frac{L}{R}$ in eq (2) we get

$$i = i_0 e^{-1} = \frac{i_0}{e} = \frac{i_0}{2.718} = 0.36\, i_0$$

So the inductive time constant of an L R circuit may also be defined as the time in which the current decays from maximum to 0.368 of the maximum value.