Subject Name: Mathematic II

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Topic : Homogeneous Function

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Homogeneous Functions

A function in which every term is of the same degree is called homogeneous function.

A function *f*(*x*,*y*) is said to be homogeneous of degree *n* if the equation

$$f(zx, zy) = z^n f(x, y)$$

Let $f(x,y) = a_0 x^n + a_1 x^{n-1} y + a_2 x^{n-2} y^2 + \ldots\ldots\ldots\ldots\ldots + a_n y^n$

be a function of x and y

Obviously each term of the function f(x,y) is of degree n. Thus f(x,y) is a homogeneous functions of degree n is x and y.

$$f(x,y) = x^n \left[a_0 + a_1 \left(\frac{y}{x} \right) + a_2 \left(\frac{y}{x} \right)^2 + \ldots\ldots\ldots\ldots\ldots + a_n \left(\frac{y}{x} \right)^n \right] = x^n F\left(\frac{y}{x} \right)$$

$$f(x,y) = y^n \left[a_0 \left(\frac{x}{y} \right)^n + a_1 \left(\frac{x}{y} \right)^{n-1} + a_2 \left(\frac{x}{y} \right)^{n-2} + \ldots\ldots\ldots\ldots\ldots + a_n \right] = y^n \phi\left(\frac{x}{y} \right)$$

Thus, every homogeneous functions of degree n in x and y can be written as either

$x^n F\left(\frac{y}{x}\right)$ or $y^n \phi\left(\frac{x}{y}\right)$

Example 1: The function $f(x,y) = xy - 2\frac{y^{5/2}}{x^{1/2}} + 5x^{3/2}y^{1/2}$ is homogeneous of degree 2.

Example 2: The function $f(x,y) = 2x + y$ is homogeneous of degree 1, since

$f(Zx, Zy) = 2(zx) + (zy) = z(2x + y)$

Euler's Theorem on Homogeneous Function

If u= f (x,y) is a homogeneous functions of degree n is x and y.

Then

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = nu$$

If u is homogeneous function in x, y and z of degree n, then

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = nu$$

An Important Deduction From Euler's Theorem

Theorem If $u = \phi(F_n)$ where F_n is a homogeneous functions of degree n, and suppose that this relation implies $F_n = f(u)$, then

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = n\frac{f(u)}{f'(u)}$$

Q. If $u = Sin^{-1}\left(\frac{x^2 + y^2}{x + y}\right)$, then show that

$$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\tan u$$

Solution We have

$$u=Sin^{-1}\left(\frac{x^2+y^2}{x+y}\right),$$

$$Sin\,u=\left(\frac{x^2+y^2}{x+y}\right)=f(x,y)$$

This f (x,y) is a homogeneous functions of degree 1, then by Euler theorem

$$x\frac{\partial f}{\partial x}+y\frac{\partial f}{\partial y}=1.f$$

$$x\frac{\partial}{\partial x}(\sin u)+y\frac{\partial}{\partial y}(\sin u)=\sin u$$

$$x\cos u\frac{\partial u}{\partial x}+y\cos u\frac{\partial u}{\partial y}=\sin u$$

$$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\tan u$$

Referential Books:

1. S.K. Sarkar, “Discrete Maths”; S. Chand & Co.,2000
2. H.K. Dass, “Advanced Engineering Mathematics”, S. Chand & Co., 9th RevisedEd
3. “Discrete Mathematics”, Schaum’s Outlines