Vector form of Faraday's Law. [Integral and differential form]

1- **Integral form** →

According to faraday's law $e = -\frac{d\phi}{dt}$ — (1)

Let us consider a closed circuit enclosing a surface S in a magnetic field $\vec{B}$ then magnetic flux through a small area $d\vec{S}$ will be $\vec{B}\cdot d\vec{S}$ and flux through the whole circuit.

$$\phi = \int_S \vec{B}\cdot d\vec{S} \quad \text{—(2)}$$

Due to change in magnetic flux an electric ~~flux~~ field induced around the circuit. By definition the line integral of the electric field gives the induced e.m.f. in the closed circuit, i.e. $e = \oint \vec{E}\cdot d\vec{l}$ — (3) Here dl is the current element of the circuit - Combining (1) (2) and (3)

$$\boxed{\oint \vec{E}\cdot d\vec{l} = -\frac{d}{dt}\int_S \vec{B}\cdot d\vec{S}} \quad \text{— (4)}$$

This is Integral form of faraday's law

2- **Differential form**.

According to stoke's law, $\oint \vec{E}\cdot d\vec{l} = \int_S (\text{Curl}\,\vec{E})\cdot d\vec{S}$ (5)

So from (4) and (5)

$$\int_S (\text{Curl}\,\vec{E})\cdot ds = -\frac{d}{dt}\int_S \vec{B}\cdot d\vec{S}$$

$$\boxed{\text{Curl}\,\vec{E} = -\frac{d\vec{B}}{dt}} \quad \text{— (6)}$$

This is differential form of faraday's law.