Subject Name: Mathematic II

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Topic: Extrema of Function's of Two Variables

By

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Maxima and Minima for functions of two variables

Let f(x,y) be a function with two independent variables x and y and let f(x,y) be continuous for all values of x and y in the small neighborhood of (a,b)

**Working Rule**

1. Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$.

2. Solve the equation $\frac{\partial f}{\partial x}=0$ and $\frac{\partial f}{\partial y}=0$ for x and y. Then pairs of values of x and y. let (a, b) be one of these pairs.

3. Find $r=\left(\frac{\partial^2 f}{\partial x^2}\right)_{(a,b)}$; $s=\left(\frac{\partial^2 f}{\partial x \partial y}\right)_{(a,b)}$ *and* $t=\left(\frac{\partial^2 f}{\partial y^2}\right)_{(a,b)}$ and calculate $rt-s^2$.

4. If $rt-s^2$ is positive and $r>0,$ then f (x,y) has a minimum at (a,b);

If $rt-s^2$ is positive and $r<0,$ then f (x,y) has a maximum at (a,b);

If $rt-s^2$ is negative, then f (x,y) has neither maximum nor minimum at (a,b);

If $rt-s^2=0$ is zero, then no conclusion can be drawn and further investigation will be required.

Q. Discuss maximum or minimum values or saddle point of the function

$f(x,y)=x^3-4xy+2y^2$

Solution Since $\frac{\partial f}{\partial x}=3x^2-4y$ and $\frac{\partial f}{\partial y}=-4x+4y.$ we have, for maxima and minima of $f$

$$\frac{\partial f}{\partial x} = 0, \ \frac{\partial f}{\partial y} = 0$$

$$3x^2 - 4y = 0,$$

$$-4x + 4y = 0$$

Solving these equations, we have

(0, 0) and $(\frac{4}{3}, \frac{4}{3})$ are critical points.

Now $r = \left(\frac{\partial^2 f}{\partial x^2}\right) = 6x;\ s = \left(\frac{\partial^2 f}{\partial x \partial y}\right) = -4\ \ and\ \ t = \left(\frac{\partial^2 f}{\partial y^2}\right) = 4$

Thus, at (0, 0)

$$rt - s^2 = 0 \times (-4) - (-4)^2 = -16 < 0$$

Hence, we conclude that (0, 0) is a saddle point of *f*.

Furthermore, at $(\frac{4}{3}, \frac{4}{3})$

r = 8, s = - 4, t = 4

$$rt - s^2 = 8 \times (4) - (4)^2 = 16 > 0$$

And r > 0.

Hence, we conclude that $(\frac{4}{3}, \frac{4}{3})$ function $f$ has minimum value. Thus, at $(\frac{4}{3}, \frac{4}{3})$ the minima value of $f_{\min}(\frac{4}{3}, \frac{4}{3}) = -\frac{32}{27}$

Q. Discuss maximum or minimum values or saddle point of the function

$$f(x,y) = x^2 + y^2 + \frac{2}{x} + \frac{2}{y}$$

Solution Since $\frac{\partial f}{\partial x} = 2x - \frac{2}{x^2}$ and $\frac{\partial f}{\partial y} = 2y - \frac{2}{y^2}$ we have, for maxima and minima of

$f$

$$\frac{\partial f}{\partial x} = 0, \ \frac{\partial f}{\partial y} = 0$$

$$2x - \frac{2}{x^2} = 0,$$

$$x = \frac{1}{x^2}$$

$$2y - \frac{2}{y^2} = 0$$

Solving this equation, we have

(1, 1) is a critical points.

Now $r = \left( \frac{\partial^2 f}{\partial x^2} \right) = 2 + \frac{4}{x^3}$; $s = \left( \frac{\partial^2 f}{\partial x \partial y} \right) = 0$ *and* $t = \left( \frac{\partial^2 f}{\partial y^2} \right) = 2 + \frac{4}{y^3}$

Thus, at (1,1)

$$rt - s^2 = 6 \times (6) - 0 = 36 > 0$$

and r =6 > 0.

Hence, we conclude that $(1,1)$ function f has minimum value. Thus, at $(1,1)$ the minima value of $f_{\min}(1,1) = 6$

Related Problems

Q1. Discuss maximum or minimum values of the function

$$f(x, y) = xy + \frac{a^3}{x} + \frac{a^3}{y}$$

Q2. Discuss maximum or minimum values of the function

$$f(x, y) = x^3 + y^3 - 3axy$$

Q3. Discuss maximum or minimum values of the function

$$f(x, y) = xy(1 - x - y)$$

**Referential Books:**


1. H.K. Dass, "Advanced Engineering Mathematics", S. Chand & Co., 9th Revised Ed
2. A.R. Vasishtha, "Differential Calculus", Krishna Pra. Media (P) Ltd.
3. V. Kumar, "Differential Calculus", Epsilon Publishing House