

Work done in isothermal reversible expansion of gases:  
Reversible isothermal expansion is an infinitely-slow increase in volume at constant temperature.  
work done in this condition is given as  $\rightarrow$

$$dW = P \times A \times dl$$

$$= P \times dV$$

The amount of work done by isothermal reversible expansion of an ideal gas from  $V_1$  to  $V_2$  is

$$W = \int_{V_1}^{V_2} P dV$$

From ideal gas equation,  $P = \frac{nRT}{V}$

Hence,

$$W = - \int_{V_1}^{V_2} \frac{nRT}{V} dV$$

$$W = -nRT \int_{V_1}^{V_2} \frac{dV}{V}$$

on integration we get

$$W = -nRT \ln \frac{V_2}{V_1}$$

Since,

$$P_1 V_1 = P_2 V_2$$

$$\text{OR } \frac{P_1}{P_2} = \frac{V_2}{V_1}$$

$$\text{OR } W = -nRT \ln \frac{P_1}{P_2}$$