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Lecture-1

Partial differentiation:-

$u = f(x, y)$.

First order derivation :- $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}$.

Second order derivative :- $\frac{\partial^2 u}{\partial x^2}, \frac{\partial^2 u}{\partial y^2}, \frac{\partial^2 u}{\partial x \partial y}$.

For example:- $u = x^2 + xy + y^2$

$$\frac{\partial u}{\partial x} = 2x + \cancel{2y}\,y.$$

$$\frac{\partial u}{\partial y} = x + 2y.$$

Now, for second order,

$$\frac{\partial^2 u}{\partial x^2} = 2.$$

$$\frac{\partial^2 u}{\partial y^2} = 2.$$

$$\frac{\partial^2 u}{\partial x \partial y} = 1.$$ { by differentiation $\partial u / \partial x$ or $\partial u / \partial y$ with respect to ∂y or ∂x }.

Third order derivative :- $\frac{\partial^3 u}{\partial x^3}, \frac{\partial^3 y}{\partial y^3}, \frac{\partial^3 u}{\partial x \partial x \partial y}$,

OR.

u_{xxx}, u_{xyy}, u_{yxy}, u_{yyx}, u_{yyy}, u_{xyx}, u_{xxy}, u_{yxx}.

Question-1 Find the first order partial derivative of.

$u(x, y) = x^y$.

$u(x, y) = x^y$

$$\frac{\partial u}{\partial x} = \cancel{0}\ y x^{y-1}$$

$$\frac{\partial u}{\partial y} = \cancel{0}.\ x^y . \log_e x. \quad \text{Ans.}$$

Question-2 Find the first order partial derivatives of $u = \tan^{-1} \dfrac{x^2+y^2}{x+y}$.

$$\frac{\partial u}{\partial x} = \frac{1}{1+\left(\frac{x^2+y^2}{x+y}\right)^2} \left[\frac{(x+y)(2x) - (x^2+y^2).(1)}{(x+y)^2}\right].$$ Ans

$$\frac{\partial u}{\partial y} = \frac{1}{1+\left(\frac{x^2+y^2}{x+y}\right)^2} \left[\frac{(x+y)(2y) - (x^2+y^2)(1)}{(x+y)^2}\right].$$ Ans

OR.

$$\frac{\partial u}{\partial x} = \frac{x^2+2xy-y^2}{(x+y)^2+(x^2+y^2)^2}.$$ {on solving it further more}.

$$\frac{\partial u}{\partial y} = \frac{y^2+2xy-x^2.}{(x+y)^2+(x^2+y^2)}$$

$\because$ the given function is symmetrical $\therefore$ we can find $\frac{\partial u}{\partial y}$ just by exchanging y with x and x with y in $\frac{\partial u}{\partial x}$.

Ans.