Q:— Customers arrive at a sales counter manned by a single person according to a poisson process with a mean rate of 20 per hour. The time required to serve a customer has an exponential distribution with a mean of 100 seconds. Find the average waiting time of a customer.

Solution:

Given that

Mean arrival rate $\lambda = 20$ per hour

& Mean service rate $\mu = \frac{60 \times 60}{100} = 36$ per hr.

The average waiting time to a customer in the queue

$$W_q = \frac{\lambda}{\mu(\mu-\lambda)} = \frac{20}{36(36-20)} \text{ hrs} = 125 \text{ seconds}$$

The average waiting time of a customer in the system

$$W_s = \frac{1}{\mu-\lambda} = \frac{1}{36-20} \text{ hr} = 225 \text{ seconds.}$$

Q.3 In a railway marshalling yard, goods trains arrive at a rate of 30 trains per day. Assuming that the inter-arrival time follows an exponential distribution and the service time distribution is also exponential with an average 36 minutes. Calculate the following:

(a) The mean queue size (line length)

(b) The probability that the queue size exceeds 10.

Solution:

$$\text{Mean arrival rate } \lambda = \frac{30}{60 \times 24} = \frac{1}{48} \text{ trains/min}$$

$$\text{Mean service rate } \mu = \frac{1}{36} \text{ trains/min}$$

$$\rho = \frac{\lambda}{\mu} = \frac{3}{4}$$

(a) $$\text{Mean queue size } (L_s) = \frac{\rho}{1-\rho} = 3 \text{ trains}$$

(b) $$\text{Probability}\left[\text{queue size} \geq 10\right] = \rho^{10}$$

$$= \left(\frac{3}{4}\right)^{10} = 0.06.$$