Q In what direction from the point $(2,1,-1)$ is the directional derivative of $f=x^2yz^3$ maximum and what is its magnitude?

$f=x^2yz^3$

$\nabla f = 2xyz^3\hat{i} + x^2z^3\hat{j} + 3x^2yz^2\hat{k}$

The directional derivative of any scalar function f at any point is the greatest in the direction of grad f.

At $(2,1,-1)$ $\nabla f = -4\hat{i} - 4\hat{j} + 12\hat{k}$

Maximum Magnitude $= \sqrt{16+16+144}$

$= 4\sqrt{11}$ Ans

Q Find the directional derivative of the function $\phi = xy^2 + yz^2 + zx^2$ along the tangent to the curve $x=t$, $y=t^2$, $z=t^3$ at $(1,1,1)$

$T = \frac{d\vec{r}}{dt}$ $\qquad$ $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$\vec{r} = t\hat{i} + t^2\hat{j} + t^3\hat{k}$

$T = \frac{d}{dt}(t\hat{i} + t^2\hat{j} + t^3\hat{k})$

$T = \hat{i} + 2t\hat{j} + 3t^2\hat{k}$

At $(1,1,1)$

$T = \hat{i} + 2\hat{j} + 3\hat{k}$

$\hat{n} = \frac{\vec{T}}{|\vec{T}|} = \frac{1}{\sqrt{14}}(\hat{i} + 2\hat{j} + 3\hat{k})$

$\phi = xy^2 + yz^2 + zx^2$

~~$\nabla\phi = (2x + 2z^2x)\hat{i} + (2y^2x + z^2)\hat{j} + (2yz + x^2)\hat{k}$~~

$\nabla\phi = (y^2 + 2zx)\hat{i} + (2xy + z^2)\hat{j} + (2yz + x^2)\hat{k}$

At $(1,1,1) \longrightarrow$

$\nabla\phi = 3\hat{i} + 3\hat{j} + 3\hat{k}$

$DD = \nabla\phi \cdot \hat{n}$

$= \frac{3}{\sqrt{14}}(1+2+3)$

$= \frac{18}{\sqrt{14}}$ Ans

Q. Find unit vector normal to the surface $z = x^2 + y^2$ at the point $(-1, -2, 5)$.

let $\phi = x^2 + y^2 - z$

$\nabla\phi = 2x\hat{i} + 2y\hat{j} - \hat{k}$

At $(-1, -2, 5) \longrightarrow$

$\nabla\phi = -2\hat{i} - 4\hat{j} - \hat{k}$

Unit Vector $= \dfrac{\nabla\phi}{|\nabla\phi|}$

$= \dfrac{1}{\sqrt{21}}(-2\hat{i} - 4\hat{j} - \hat{k})$ Ans