Ques:- Verify divergence thm. for $\vec{F} = (x^2 - yz)\hat{i} + (y^2 - zx)\hat{j} + \ldots$

taken over the rectangular parallelopiped ~~0≤a, 0≤~~
$0 \le x \le a,\ 0 \le y \le b,\ 0 \le z \le c.$

By Gauss-Divergence thm, we have

$$\int_V \nabla \vec{F}\, dv = \int_S \vec{F} \cdot \hat{n}\, ds.$$

$$\nabla \vec{F} = 2x + 2y + 2z.$$

$$L.H.S = \int_0^a \int_0^b \int_0^c 2(x+y+z)\, dz\, dy\, dx.$$

$$2\int_0^a \int_0^b \int_0^c \left[xz + yz + \frac{z^2}{2}\right]_0^c dy\, dx$$

$$2\int_0^a \int_0^b \left[xc + yc + \frac{c^2}{2}\right] dy\, dx$$

$$2\int_0^a \left[xcy + \frac{y^2}{2}c + \frac{yc^2}{2}\right]_0^b dy\, dx$$

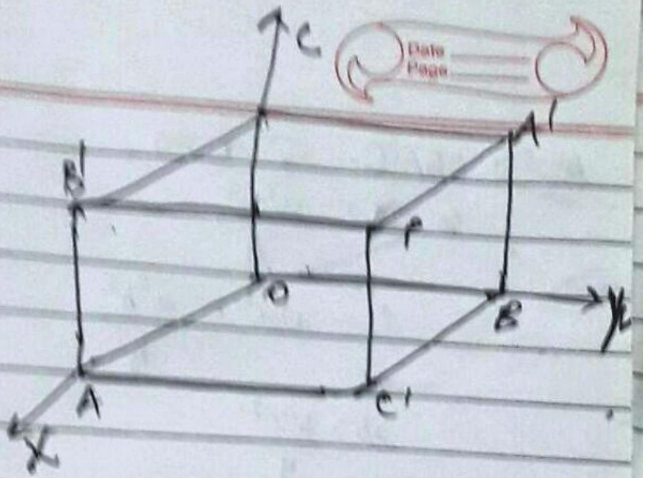
$$\left[xbc + \frac{b^2}{2}c + \frac{bc^2}{2}\right]$$

$$2 \left[\frac{x^2}{2}bc + \frac{xb^2c}{2} + \frac{xbc^2}{2}\right]_0^a$$

$$2\left[\frac{a^2bc}{2} + \frac{ab^2c}{2} + \frac{abc^2}{2}\right]$$

$$\boxed{abc\,(a+b+c)}$$

R.H.S S_1 :- $OAC'B$
S_2 :- $CB'PA'$
S_3 :- $OBA'C$
S_4 :- $AC'PB'$
S_5 :- $OCB'A$
S_6 :- $BA'PC'$



At S_1 :- $OAC'B$, $z = 0$, $dz = 0$.

$\cancel{\iint abc(a+b+c)\,c\,dy}$

$$\int_S \vec{F}\cdot\hat{n}\,dS = \int_0^a\int_0^b \left[(x^2-yz)\hat{i} + (y^2-zx)\hat{j} + (z^2-xy)\hat{k}\right]\left[(-\hat{k})\right] dy\,dx$$

$$\int_0^a\int_0^b (-z^2+xy)\,dy\,dx$$

$$\int_0^a \left[z^2y + \frac{xy^2}{2}\right]_0^b dx$$

$$\left[z^2b + \frac{xb^2}{2}\right] dx$$

$$\int_0^a \left[+\frac{xb^2}{2}\right] dx$$

$$\left[+\frac{x^2b^2}{4}\right] = \boxed{+\frac{a^2b^2}{4}}$$

At S_2 :- $CB'PA'$.

$z = c$, $dz = 0$.

$$\int_S \vec{F}\cdot\hat{n}\,dS = \left[(x^2-yz)\hat{i} + (y^2-zx)\hat{j} + (z^2-xy)\hat{k}\right]\left[\hat{k}\right] dy\,dx$$

$$= \iint \left[z^2 - xy\right] dy\,dx$$

$$\int_0^a \left[z^2y - \frac{xy^2}{2}\right]_0^b dx$$

$$\int_0^a \left[c^2b - \frac{xb^2}{2}\right] dx$$

$$= \boxed{c^2ba - \frac{a^2b^2}{4}}$$

A33 QBA(Q. $\ldots$

$S_1 = \frac{a^2b^2}{4}$

$S_2 = abc^2 - \frac{a^2b^2}{4}$

$S_3 = \frac{b^2c^2}{4}$

$S_4 = bca^2 - \frac{b^2c^2}{4}$

$S_5 = \frac{a^2c^2}{4}$

$S_6 = acb^2 - \frac{a^2c^2}{4}$

$$\cancel{\frac{a^2b^2}{4}} + abc^2 - \cancel{\frac{a^2b^2}{4}} + \cancel{\frac{b^2c^2}{4}} + bca^2 - \cancel{\frac{b^2c^2}{4}} + \cancel{\frac{a^2c^2}{4}} + acb^2 - \ldots$$

$$= abc(c + a + b)$$

$$= \boxed{abc(a + b + c)}$$