Rules to find the extreme values of a function $z = f(x, y)$:-

$\frac{\partial z}{\partial x} = ?$, $\frac{\partial z}{\partial y} = ?$

$\underbrace{\frac{\partial^2 z}{\partial x^2}}_{r} = ?$, $\underbrace{\frac{\partial^2 z}{\partial y^2}}_{t} = ?$, $\underbrace{\frac{\partial^2 z}{\partial x \partial y}}_{s}$

$rt - s^2 = ?$

If $rt - s^2 > 0$, and $r > 0$ then f has minimum value.

If $rt - s^2 > 0$, and $r < 0$ then f has maximum value.

If $rt - s^2 < 0$, then z has no exteme value.

Q- Find the extreme value of function $x^3 + y^3 - 3axy$.

<u>Solution</u> :- let $f(x, y) = x^3 + y^3 - 3axy$

$f_x = 3x^2 - 3ay = 0 \Rightarrow y = x^2/a$

$f_y = 3y^2 - 3ax = 0$

$y^2 - ax = 0$

$\frac{x^4}{a^2} - ax = 0 \Rightarrow x(x^3 - a^3) = 0$

$x(x-a)(x^2 + a^2 - ax) = 0$

$x = 0, a$

when $x = 0 \Rightarrow y = 0$

$x = a \Rightarrow y = a$

$\therefore$ There are two stationary points $(0, 0)$ and (a, a).

Now, $rt - s^2 = 36xy - 9a^2$

At $(0,0)$, $rt - s^2 = -9a^2 < 0$.

$\Rightarrow$ There is no exterme value at $(0,0)$.

At (a,a) $rt - s^2 = 27a^2 > 0$

$\Rightarrow$ f has extreme value at (a, a).

Now, $r = 6a$

If $a > 0$, $r > 0$ so that $f(x,y)$ has a minimum value at (a,a).

Min value $= a^3 + a^3 - 3a^2 = -a^3$.

If $a < 0$, $r > 0$, so that f has maximum value at (a,a).

Max value $= -a^3 - a^3 + 3a^3 = a^3$.