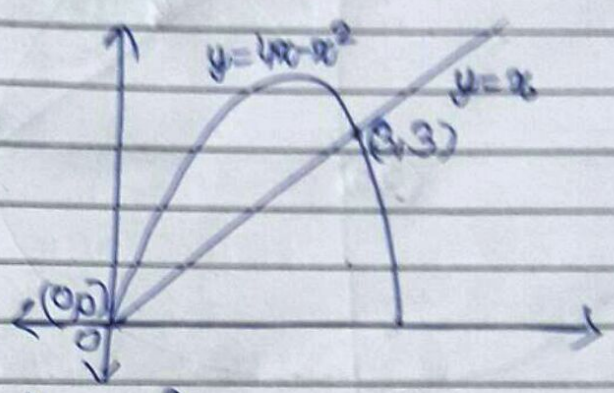
## Area by Double Integrations-

$$A = \int_a^b \int_{\phi_1(y)}^{\phi_2(y)} dx\,dy \quad \text{or} \quad A = \int_a^b \int_{\phi_1(x)}^{\phi_2(x)} dy\,dx$$

**Q.** Find the area lying between the parabola $y = 4x - x^2$ and the line $y = x$.




$x = 4x - x^2$

$x^2 - 3x = 0$

$x(x-3) = 0$

$x = 0, 3$

$$A = \int_0^3 \int_x^{4x-x^2} dy\,dx$$

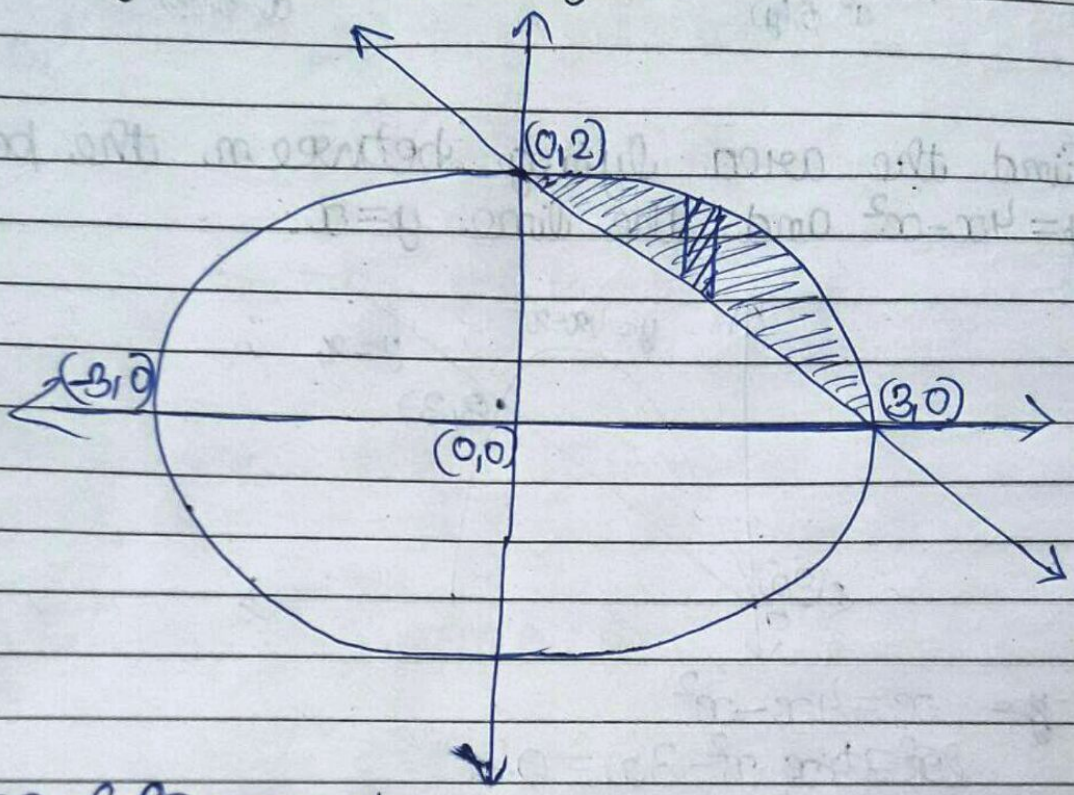
$$A = \int_0^3 (4x - x^2 - x)\,dx$$

$$A = \left[\frac{3x^2}{2} - \frac{x^3}{3}\right]_0^3$$

$$A = \frac{27}{2} - \frac{27}{3}$$

$$A = \frac{27}{6} \times 1 = \frac{9}{2} \text{ Ans}$$

Q. Find the smaller of the areas bounded by the ellipse $4x^2+9y^2=36$ and the straight line $2x+3y=6$.




~~$A=\int\int dy\,dx\,dy$~~

~~$A=\int_0^{6-2x/3}\int dy\,dx$~~

$$A=\int_0^3\int_{\frac{6-2x}{3}}^{\frac{1}{3}\sqrt{36-4x^2}} dy\,dx$$

$$A=\int_0^3 \frac{1}{3}\left[\sqrt{36-4x^2}-(6-2x)\right]dx$$

$$=\frac{2}{3}\int_0^3\left[\sqrt{9-x^2}-(3-x)\right]dx$$

$$=\frac{2}{3}\left[\frac{x\sqrt{9-x^2}}{2}+\frac{9}{2}\sin^{-1}\frac{x}{3}-3x+\frac{x^2}{2}\right]_0^3$$

$$=\frac{2}{3}\left(\frac{9}{2}\times\frac{\pi}{2}-9+\frac{9}{2}\right)$$

$$=\frac{2}{3}\left(\frac{9\pi}{4}+\frac{9}{2}\right)=\frac{3}{2}\left(\frac{\pi}{2}+1\right)$$

$$=\frac{2}{3}\times\frac{9}{2\times2}(\pi-2)$$

$$=\frac{3}{2}(\pi-2)$$ Ans