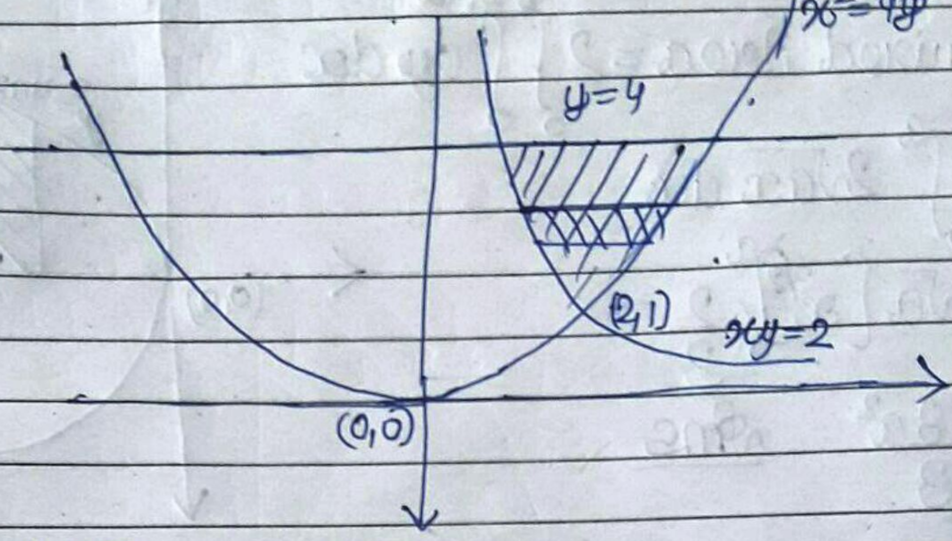
Q. Determine the area of region bounded by the curves $xy=2$ $4y=x^2$, $y=4$.



$$A = \int_{1}^{4}\int_{2/y}^{2\sqrt{y}} dx\,dy$$

$$A = \int_{1}^{4}\left(2\sqrt{y} - \frac{2}{y}\right)dy$$

$$= \cancel{2\left[2y^{1/2} - 2\log 2y\right]_1^4}$$

$$= \cancel{2(4 - \log 4)} -$$

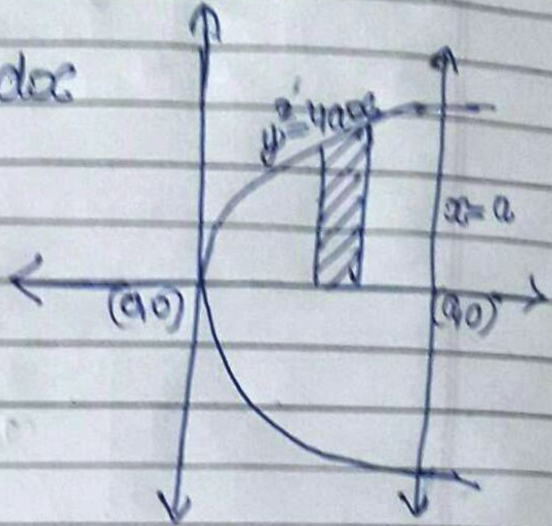
$$= 2\left[y^{3/2}\times\frac{2}{3} - \log y\right]_1^4$$

$$A = 2\left[\left(\frac{16}{3} - \log 4\right) - \left(\frac{2}{3}\right)\right]$$

$$A = \frac{28}{3} - 4\log 2 \quad \text{Ans}$$

Q. Find the area bounded by the parabola $y^2 = 4ax$ and its latus rectum.

Required Area $= 2\int_0^a \int_0^{2\sqrt{ax}} dy\,dx$

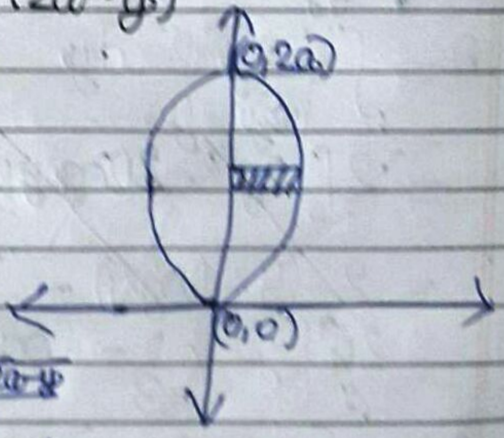


$= 2\int_0^a 2\sqrt{ax}\,dx$

$= 4\sqrt{a}\left[x^{3/2}\right]_0^a \times \frac{2}{3}$

$= \frac{8a^2}{3}$ <u>Ans</u>

Q. By double integration find the whole area of the curve $a^2x^2 = y^3(2a-y)$



Required Area $= 2\int_0^{2a}\int_0^{\frac{y^{3/2}\sqrt{2a-y}}{a}} dx\,dy$

$= \frac{2}{a}\int_0^{2a} y^{3/2}\sqrt{2a-y}\,dy$

Put $y = 2a\sin^2\theta$

$dy = 4a\sin\theta\cos\theta\,d\theta$

$= \frac{2}{a}\int_0^{2a} a^{3/2}\cdot 2^{3/2}\sqrt{2a}\cos\theta\cdot 4a\sin\theta\cos\theta\,d\theta$

$= \pi a^2$