Q. If $x = u(1+v)$ $y = v(1+u)$ then find the value of $\frac{\partial(x,y)}{\partial(u,v)}$

$x = u(1+v)$ $y = v(1+u)$

$x = u + uv$ $y = v + uv$

$\frac{\partial x}{\partial u} = 1+v$ $\frac{\partial y}{\partial u} = v$

$\frac{\partial x}{\partial v} = u$ $\frac{\partial y}{\partial v} = 1+u$

$$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix}$$

$$= \begin{vmatrix} 1+v & u \\ v & 1+u \end{vmatrix}$$

$$= 1+u+v+uv-uv$$

$$= 1+u+v \quad \text{Ans}$$

Q. If $x = uv$, $y = \frac{u+v}{u-v}$ then find the value of $\frac{\partial(u,v)}{\partial(x,y)}$

$\frac{\partial x}{\partial u} = v$, $\frac{\partial x}{\partial v} = u$

$$\frac{\partial y}{\partial u} = \frac{u-v-(u+v)}{(u-v)^2} = \frac{-2v}{(u-v)^2}$$

$$\frac{\partial y}{\partial v} = \frac{(u-v)+(u+v)}{(u-v)^2} = \frac{2u}{(u-v)^2}$$

$$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix}$$

$$= \begin{vmatrix} v & u \\ \frac{-2v}{(u-v)^2} & \frac{2u}{(u-v)^2} \end{vmatrix}$$

$$= \frac{4uv}{(u-v)^2}$$

$$\frac{\partial(u,v)}{\partial(x,y)} = \frac{1}{\frac{\partial(x,y)}{\partial(u,v)}}$$

$$= \frac{(u-v)^2}{4uv} \quad \underline{\text{Ans}}$$

Q If $u = x(1-y)$, $v = xy$ find $\rightarrow \frac{\partial(u,v)}{\partial(x,y)}$

$$\frac{\partial(u,v)}{\partial(x,y)} = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix}$$

$$= \begin{vmatrix} 1-y & -x \\ y & x \end{vmatrix}$$

$$= x - xy + xy$$

$$= x \quad \underline{\text{Ans}}$$