Q If $u = \frac{x}{y-z}$, $v = \frac{y}{z-x}$, $w = \frac{z}{x-y}$

Find → $\frac{\partial(u,v,w)}{\partial(x,y,z)}$

$$\frac{\partial(u,v,w)}{\partial(x,y,z)} = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z} \end{vmatrix}$$

$$= \begin{vmatrix} \frac{1}{y-z} & \frac{-x}{(y-z)^2} & \frac{x}{(y-z)^2} \\ \frac{y}{(z-x)^2} & \frac{1}{z-x} & \frac{-y}{(z-x)^2} \\ \frac{-z}{(x-y)^2} & \frac{z}{(x-y)^2} & \frac{1}{x-y} \end{vmatrix}$$

Taking $\frac{1}{(y-z)^2}$, $\frac{1}{(z-x)^2}$ $\frac{1}{(x-y)^2}$ common from R_1 R_2 & R_3 respectively ⟶

$$= \begin{vmatrix} 1 & \frac{-x}{y-z} & \frac{-x}{y-z} \\ 0 & \left(1+\frac{x}{y-z}\right) & \left(\frac{-y}{z-x} - \frac{x}{y-z}\right) \\ 0 & \left(\frac{z}{x-y} - 1\right) & \left(1+\frac{y}{z-x}\right) \end{vmatrix}$$

$$= \left(1+\frac{x}{y-z}\right)\left(1+\frac{y}{z-x}\right) - \left[\left(\frac{y}{z-x}\right) + \frac{x}{y-z}\right]\left(\frac{z}{x-y} - 1\right)$$

$$= 1 + \cancel{\frac{y}{z-x}} + \cancel{\frac{x}{y-z}} + \frac{xy}{(y-z)(z-x)} + \frac{yz}{(z-x)(x-y)} - \cancel{\frac{y}{z-x}}$$

$$+ \frac{xz}{(y-z)(x-y)} - \cancel{\frac{x}{y-z}}$$

$$= \frac{xy(x-y) + yz(y-z) + zx(z-x)}{(x-y)(y-z)(z-x)} + 1$$

$$= x^2y - xy^2 + y^2z - yz^2 + z^2x - x^2z$$

$$= \frac{1}{(x-y)^2(y-z)^2(z-x)^2}\begin{vmatrix} y-z & -x & -x \\ y & z-x & -y \\ -z & z & x-y \end{vmatrix}$$

$$= \frac{1}{(x-y)^2(y-z)^2(z-x)^2}\left[\begin{array}{l}(y-z)(zx-\cancel{yz}-x^2+xy+\cancel{yz})+ \\ x(xy-y^2+yz+yz+z^2+zx)\end{array}\right]$$

$$= \frac{1}{(x-y)^2(y-z)^2(z-x)^2}\left(\begin{array}{l}\cancel{xyz}-\cancel{x^2y}+\cancel{xy^2}-\cancel{xz^2}-\cancel{x^2z}-\cancel{xyz} \\ +\cancel{x^2y}-\cancel{xy^2}-\cancel{xyz}+\cancel{xyz}+\cancel{xz^2}+\cancel{x^2z}\end{array}\right)$$

$= 0$ Ans

Q. If $u^3+v^3 = x+y$ $\quad u^2+v^2 = x^3+y^3$ then —
Show that $\rightarrow$ $\dfrac{\partial(u,v)}{\partial(x,y)} = \dfrac{y^2-x^2}{2uv(u-v)}$

$$\frac{\partial(u,v)}{\partial(x,y)} = (-1)^2 \left[\frac{\dfrac{\partial(F_1,F_2)}{\partial(x,y)}}{\dfrac{\partial(F_1,F_2)}{\partial(u,v)}}\right]$$

$$F_1 = u^3+v^3-x-y$$
$$F_2 = u^2+v^2-x^3-y^3$$

$$\cancel{\frac{\partial(u,v)}{\partial(x,y)}} \quad \frac{\partial(F_1,F_2)}{\partial(x,y)} = \begin{vmatrix} \dfrac{\partial F_1}{\partial x} & \dfrac{\partial F_1}{\partial y} \\ \dfrac{\partial F_2}{\partial x} & \dfrac{\partial F_2}{\partial y} \end{vmatrix}$$

$$= \begin{vmatrix} -1 & -1 \\ -3x^2 & -3y^2 \end{vmatrix}$$

$$= 3y^2 - 3x^2$$
$$= 3(y^2-x^2)$$

$$\frac{\partial(F_1,F_2)}{\partial(u,v)} = \begin{vmatrix} 3u^2 & 3v^2 \\ 2u & 2v \end{vmatrix}$$

$$= 6u^2v - 6uv^2$$
$$= 6uv(u-v)$$

$$\frac{\partial(u,v)}{\partial(x,y)} = \frac{y^2-x^2}{2uv(u-v)}$$