

14.02.23

## Lecture - 9

Question-1 If  $u = x^2 - y^2 + \sin yz$ , where  $y = e^x$  and  $z = \log x$ , find  $\frac{du}{dx}$ .

$$\frac{du}{dx} = \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} \frac{dy}{dx} + \frac{\partial u}{\partial z} \frac{dz}{dx}$$

$$\frac{\partial u}{\partial x} = 2x$$

$$\frac{\partial u}{\partial y} = -2y + z \cos yz$$

$$\frac{\partial u}{\partial z} = y \cos yz$$

$$\frac{dy}{dx} = e^x$$

$$\frac{dz}{dx} = \frac{1}{x}$$

$$\frac{du}{dx} = 2x + (-2y)e^x + \frac{y \cos yz}{x}$$

$$= 2x - 2e^{2x} + \frac{e^x \cos(e^x \log x)}{x}$$

$$= 2(x - e^{2x}) + e^x \cos(e^x \log x)$$

$$\frac{du}{dx} = 2x - ((2y) + z \cos yz) \cdot e^x + \frac{y \cos yz}{x}$$

$$= 2x - 2e^{2x} + e^x \log x \cos(e^x \log x) + \frac{e^x}{x} \cos(e^x \log x)$$

$$= 2(x - e^x) + e^x \cos(e^x \log x) \left( \log x + \frac{1}{x} \right)$$

Question-2 If  $u = f(r, s)$  and  $r = x + y$ ,  $s = x - y$  show that  $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 2 \frac{\partial u}{\partial r}$ .

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial r} \cdot \frac{\partial r}{\partial x} + \frac{\partial u}{\partial s} \frac{ds}{dx}$$

$$= \frac{\partial u}{\partial r} \cdot 1 + \frac{\partial u}{\partial s} \cdot 1$$

$$= \frac{\partial u}{\partial r} + \frac{\partial u}{\partial s} = 1$$



$$\frac{\partial u}{\partial y} = \frac{\partial u}{\partial s} \frac{ds}{dy} + \frac{\partial u}{\partial t} \frac{dt}{dy}$$

$$= \frac{\partial u}{\partial s} + \frac{\partial u}{\partial t} (-1)$$

$$\frac{\partial u}{\partial y} = \frac{\partial u}{\partial s} - \frac{\partial u}{\partial t} \quad \text{--- 2.}$$

Adding eq. 1 and eq. 2.

$$= \frac{\partial u}{\partial s} + \frac{\partial u}{\partial y}$$

$$= \frac{\partial u}{\partial s} + \frac{\partial u}{\partial s} + \frac{\partial u}{\partial s} - \frac{\partial u}{\partial s}$$

$$= \frac{2 \partial u}{\partial s}$$

∴ - RHS

Hence Proved. Ans.

Question - 3 If  $z$  is a function of  $x$  and  $y$ , where  $x = e^u + e^{-v}$  and  $y = e^{-u} - e^v$ . Show that

$$\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = x \frac{\partial z}{\partial x} - y \frac{\partial z}{\partial y}$$

$$\text{Now, } \frac{\partial z}{\partial u} = \frac{\partial z}{\partial x} \frac{\partial x}{\partial u} + \frac{\partial z}{\partial y} \frac{\partial y}{\partial u}$$

$$= \frac{\partial z}{\partial x} \cdot e^u + \frac{\partial z}{\partial y} e^{-u} (-1)$$

$$= e^u \frac{\partial z}{\partial x} - e^{-u} \frac{\partial z}{\partial y} \quad \text{--- 1.}$$

$$\frac{\partial z}{\partial v} = \frac{\partial z}{\partial x} \frac{\partial x}{\partial v} + \frac{\partial z}{\partial y} \frac{\partial y}{\partial v}$$

$$= \frac{\partial z}{\partial x} e^{-v} (-1) + \frac{\partial z}{\partial y} (-e^v)$$

$$= -\frac{\partial z}{\partial x} e^{-v} - \frac{\partial z}{\partial y} e^v \quad \text{--- 2.}$$

on subtracting 1. and 2.

$$\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = e^u \frac{\partial z}{\partial x} - e^{-u} \frac{\partial z}{\partial y} + e^{-v} \frac{\partial z}{\partial x} + e^v \frac{\partial z}{\partial y}$$



$$= \frac{\partial z}{\partial x} (e^u + e^{-v}) - \frac{\partial z}{\partial y} (e^{-u} - e^v)$$

$$\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y} = x \frac{\partial z}{\partial x} - y \frac{\partial z}{\partial y}$$

∴ RHS

Hence Proved. Ans.

Question-4 If  $f(x, y) = 0$ ,  $g(y, z) = 0$ , show that

$$\frac{\partial f}{\partial y} \cdot \frac{\partial g}{\partial z} \cdot \frac{\partial z}{\partial x} = \frac{\partial f}{\partial x} \cdot \frac{\partial g}{\partial y}$$

$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial y} \cdot \frac{dy}{dx}$$

$$\frac{dy}{dx} = - \frac{\partial f / \partial x}{\partial f / \partial y}$$

$$\frac{\partial z}{\partial x} = - \frac{\partial g / \partial y}{\partial g / \partial z}$$

Now,

∴

$$\frac{\partial f}{\partial y} \cdot \frac{\partial g}{\partial z} \cdot \frac{\partial z}{\partial x} = \frac{\partial f}{\partial y} \cdot \frac{\partial g}{\partial z} \cdot \frac{dy}{dx}$$

$$= \frac{\partial f}{\partial y} \cdot \frac{\partial g}{\partial z} \cdot \left( - \frac{\partial f / \partial x}{\partial f / \partial y} \right) \times \left( - \frac{\partial g / \partial y}{\partial g / \partial z} \right)$$

$$= \frac{\partial f}{\partial y} \cdot \frac{\partial g}{\partial z} \cdot \left( \frac{\partial f / \partial x}{\partial f / \partial y} \right) \times \left( \frac{\partial g / \partial y}{\partial g / \partial z} \right)$$

$$= \frac{\partial f}{\partial x} \cdot \frac{\partial g}{\partial y}$$

∴ RHS

Hence Proved.