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Q) If $y_1 = 1 - x_1$, $y_2 = x_1(1 - x_2)$, $y_3 = x_1 x_2 (1 - x_3)$ and so on $y_n = x_1 x_2 \dots x_{n-1}(1 - x_n)$, then show that $\frac{\partial(y_1, y_2 \dots y_n)}{\partial(x_1, x_2 \dots x_n)} = (-1)^n x_1^{n-1} x_2^{n-2} \dots x_{n-1}$.

Ans: We have $\frac{\partial(y_1, y_2 \dots y_n)}{\partial(x_1, x_2, \dots x_n)} = \frac{\partial y_1}{\partial x_1} \cdot \frac{\partial y_2}{\partial x_2} \dots \frac{\partial y_n}{\partial x_n}$

$= -1 (-x_1)(-x_1 x_2) \dots (-x_1 x_2 \dots x_{n-1})$

$\Rightarrow (-1)^n (x_1^{n-1}) \cdot x_2^{n-2} \dots x_{n-1}$

Q) If $y_1 = \cos x_1$, $y_2 = \sin x_1 \cos x_2$, $y_3 = \sin x_1 \sin x_2 \cos x_3$, then show that $\frac{\partial(y_1, y_2, y_3)}{\partial(x_1, x_2, x_3)} = -\sin^3 x_1 \sin^2 x_2 \cdot \sin x_3$

Ans: W.K.T, $\frac{\partial(y_1, y_2, y_3)}{\partial(x_1, x_2, y_3)} = \frac{\partial y_1}{\partial x_1} \cdot \frac{\partial y_2}{\partial x_2} \cdot \frac{\partial y_3}{\partial x_3}$

$\Rightarrow -\sin x_1 \cdot (\sin x_1 \cdot (-\sin x_2) + \cancel{\cos x_2} \cdot \cancel{3} - \cancel{3} \sin x_1 \sin x_2 - \sin x_3$

$\Rightarrow -\sin^3 x_1 \cdot \sin^2 x_2 \cdot \sin x_3$

Q) If $u = xyz$, $v = x^2 + y^2 + z^2$, $w = x + y + z$ find the Jacobian of $\frac{\partial(x, y, z)}{\partial(u, v, w)}$,

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Ans $\frac{\partial(\cancel{y}\, x, y, z)}{\partial(u, v, w)} = \frac{\partial x}{\partial u} \cdot \frac{\partial y}{\partial v} \cdot \frac{\partial z}{\partial w}$

But here ~~u, v, w~~ x, y, z are function of u, v, w

So, $\frac{\partial(u, v, w)}{\partial(x, y, z)} = \cancel{\frac{\partial u}{\partial x} \cdot \frac{\partial v}{\partial y}} \cdot \frac{\partial w}{\partial z}$

$$\Rightarrow \cancel{yz \cdot 2y \cdot x} = \begin{vmatrix} \partial u/\partial x & \partial u/\partial y & \partial w/\partial z \\ \partial v/\partial x & \partial v/\partial y & \partial w/\partial z \\ \partial w/\partial x & \partial w/\partial y & \partial w/\partial z \end{vmatrix}$$

$$\Rightarrow \begin{vmatrix} yz & xz & xy \\ 2x & 2y & 2z \\ 1 & 1 & 1 \end{vmatrix} \quad C_1 \rightarrow C_1 - C_2$$

$$\Rightarrow \begin{vmatrix} z(y-x) & xz & xy \\ 2(x-y) & 2y & 2z \\ 0 & 1 & 1 \end{vmatrix} \quad C_2 \rightarrow C_2 - C_3$$

$$\Rightarrow \begin{vmatrix} z(y-x) & x(z-y) & xy \\ 2(x-y) & 2(y-z) & 2z \\ 0 & 0 & 1 \end{vmatrix}$$

$z(y-x)[2(y-z)] - x(z-y)[2(x-y)] \cancel{+xy}$

$\Rightarrow z(y-x)(2y-2z) - [xz - xy][2x-2y]$

$\Rightarrow (zy - zx)(2y - 2z) - [2x^2z - 2xyz - 2x^2y + 2y^2x]$

$\Rightarrow 2zy^2 - 2z^2y - 2xyz + 2z^2x - 2x^2z + 2xy + 2x^2y - 2y^2x$

$\Rightarrow 2zy^2 - 2z^2y - 2xyz$

$2y^2(z-x) - 2z^2(y-x) - 2x^2(z-y)$

$= 2y^2(z-x) - 2z^2(y-x) - 2x^2(z-y)$