DATE

Sol^n:- $$\frac{\partial(u,v,w)}{\partial(x,y,z)} = \begin{vmatrix} \partial u/\partial x & \partial u/\partial y & \partial u/\partial z \\ \partial v/\partial x & \partial v/\partial y & \partial v/\partial z \\ \partial w/\partial x & \partial w/\partial y & \partial w/\partial z \end{vmatrix}$$

$$= \begin{vmatrix} yz & xz & xy \\ 2x & 2y & 2z \\ 1 & 1 & 1 \end{vmatrix}$$

$C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$

$$= \begin{vmatrix} yz & z(x-y) & y(x-z) \\ 2x & 2(y-x) & 2(z-x) \\ 1 & 0 & 0 \end{vmatrix}$$

$= -z(x-y)(-2(z-x)) + y(x-z)(-2(y-x))$

$= 2z(x-y)(z-x) - 2y(x-z)(y-x)$

$= 2(x-y)(z-x)[z-y]$

$= -2(x-y)(y-z)(z-x)$

By chain rule

$J_1 \cdot J_2 = 1$

$$J \quad \frac{\partial(x,y,z)}{\partial(u,v,w)} = \frac{-1}{2(x-y)(y-z)(z-x)}$$

Q- If $x+y+z = u$, $y+z = uv$, $z = uvw$

then show that $\frac{\partial(x,y,z)}{\partial(u,v,w)} = u^2 v$

$x = u - (y+z)$, $y = uv - z$

$x = u - uv$

$x = u(1-v)$, $y = uv - uvw$

$z = uvw$ $y = uv(1-w)$

$$\frac{\partial(x,y,z)}{\partial(u,v,w)} = \begin{vmatrix} \partial x/\partial u & \partial x/\partial v & \partial x/\partial w \\ \partial y/\partial u & \partial y/\partial v & \partial y/\partial w \\ \partial z/\partial u & \partial z/\partial v & \partial z/\partial w \end{vmatrix}$$

$$= \begin{vmatrix} (1-v) & -u & 0 \\ v(1-w) & u(1-w) & -uv \\ vw & uw & uv \end{vmatrix}$$

$$= \begin{vmatrix} (1-v) & -u & 0 \\ v & u\,(\cancel{\quad}) & 0 \\ vw & uw & uv \end{vmatrix}$$

$$= (1-v)\,\big(u^2(1+w)v\big) + u\,(v^2u)$$

$$= (1-v)\,vu^2(1+w) + u^2v^2$$

$$=$$

$$= (1-v)(u^2v) + u(uv^2)$$

$$= u^2v - u^2v^2 + u^2v^2$$

$$= u^2v$$

Q- If $u^3+v^3+w^3 = x+y+z$, $u^2+v^2+w^2 = x^3+y^3+z^3$
$u+v+w = x^2+y^2+z^2$ then show that

$$\frac{\partial(u,v,w)}{\partial(x,y,z)} = \frac{(x-y)(y-z)(z-x)}{(u-v)(v-w)(w-u)}$$

Soln :-

$$F_1 = u^3+v^3+w^3-x-y-z$$
$$F_2 = u^2+v^2+w^2-x^3-y^3-z^3$$
$$F_3 = u+v+w-x^2-y^2-z^2$$

$$\frac{\partial(u,v,w)}{\partial(x,y,z)} = (-1)^3 \frac{\dfrac{\partial(F_1,F_2,F_3)}{\partial(x,y,z)}}{\dfrac{\partial(F_1,F_2,F_3)}{\partial(u,v,w)}}$$

$$\frac{\partial(F_1, F_2, F_3)}{\partial(x, y, z)} = \begin{vmatrix} \partial F_1/\partial x & \partial F_1/\partial y & \partial F_1/\partial z \\ \partial F_2/\partial x & \partial F_2/\partial y & \partial F_2/\partial z \\ \partial F_3/\partial x & \partial F_3/\partial y & \partial F_3/\partial z \end{vmatrix}$$

$$= \begin{vmatrix} -1 & -1 & -1 \\ -3x^2 & -3y^2 & -3z^2 \\ -2x & -2y & -2z \end{vmatrix}$$

$C_1 \to C_1 - C_2$ and $C_3 \to C_3 - C_2$

$$= \begin{vmatrix} 0 & -1 & 0 \\ 3(y^2-x^2) & -3y^2 & 3(y^2-z^2) \\ 2(y-x) & -2y & 2(y-z) \end{vmatrix}$$

$$= 1\left(6(y-x)(y+x)(y-z) - 6(y-z)(y+z)(y-x)\right)$$

$$= 6(y-x)(y-z)\left[y+x-y-z\right]$$

$$6(y-x)(y-z)(x-z)$$

$$\frac{\partial(f_1, f_2, f_3)}{\partial(u, v, w)} = \begin{vmatrix} \partial f_1/\partial u & \partial f_1/\partial v & \partial f_1/\partial w \\ \partial f_2/\partial u & \partial f_2/\partial v & \partial f_2/\partial w \\ \partial f_3/\partial u & \partial f_3/\partial v & \partial f_3/\partial w \end{vmatrix}$$

$$= \begin{vmatrix} 3u^2 & 3v^2 & 3w^2 \\ 2u & 2v & 2w \\ 1 & 1 & 1 \end{vmatrix}$$

$C_1 \to C_1 - C_2$ and $C_3 \to C_3 - C_2$.

$$= \begin{vmatrix} 3(u^2-v^2) & 3v^2 & 3(w^2-v^2) \\ 2(u-v) & 2v & 2(w-v) \\ 0 & 1 & 0 \end{vmatrix}$$

PAGE

DATE

$$= -1\left(6(u-v)(u+v)(w-v) - 6(w-v)(w+v)(u-v)\right)$$

$$= -6(u-v)(w-v)\left[u+\cancel{v}-w-\cancel{v}\right]$$

$$-6(u-v)(w-v)(u-w)$$

$$= 6(u-v)(v-w)(u-w)$$

$$\frac{\partial(u,v,w)}{\partial(x,y,z)} = (-1)^3 \frac{6(x-y)(y-z)(z-x)}{6(u-v)(v-w)(w-u)}$$

$$= \frac{(-1)(x-y)(y-z)(z-x)}{(-1)(u-v)(v-w)(w-u)}$$

$$\frac{\partial(u,v,w)}{\partial(x,y,z)} = \frac{(x-y)(y-z)(z-x)}{(u-v)(v-w)(w-u)}$$

Q- If u,v,w are the roots of the eq^n $(x-a)^3+(y-b)^3+(z-c)^3=0$ then find $\frac{\partial(u,v,w)}{\partial(a,b,c)}$

If u,v,w are the roots of eq^n.
$(x-a)^3+(y-b)^3+(z-c)^3=0$.

$$3x^3 - 3x^2(a+b+c) + 3x(a^2+b^2+c^2) - (a^3+b^3+c^3) = 0$$

$$u+v+w = \frac{3(a+b+c)}{3} = (a+b+c)$$

$$uv+vw+wu = \frac{3(a^2+b^2+c^2)}{3} = (a^2+b^2+c^2)$$

$$uvw = \frac{(a^3+b^3+c^3)}{3}$$

$$F_1 = u+v+w-a-b-c$$

$$F_2 = uv+vw+wu-a^2-b^2-c^2$$

$$F_3 = \frac{3uvw - a^3-b^3+c^3}{\cancel{3}}$$