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Question-1 Examine for minimum or maximum values of $\sin x + \sin y + \sin(x+y)$

Let $f(x,y) = \sin x + \sin y + \sin(x+y)$

$\frac{\partial f}{\partial x} = \cos x + \cos(x+y) = 0.$

$\frac{\partial f}{\partial y} = \cos y + \cos(x+y) = 0.$

From above,

$\cos x + \cos(x+y) = 0$ ------ 1.

$\cos y + \cos(x+y) = 0$ ------ 2.

on subtracting 1. from 2.

$\cos x - \cos y = 0$

$\cos x = \cos y.$

$x = y.$

on putting these values in above equation,

$\cos x + \cos 2x = 0.$

$\cos 2x = -\cos x.$

$\cos 2x = \cos(\pi - x)$

$3x = \pi.$

$x = \pi/3, \quad y = \pi/3.$

Now,

$\frac{\partial^2 f}{\partial x^2} = -\sin x - \sin(x+y)$

$\frac{\partial^2 f}{\partial y^2} = -\sin y - \sin(x+y)$

$\frac{\partial^2 f}{\partial x \partial y} = -\sin(x+y)$

Now at point $(\pi/3, \pi/3)$

$rt - s^2 = -\sqrt{3}(-\sqrt{3}) - \left(-\frac{\sqrt{3}}{2}\right)^2$

$= \cancel{\frac{12}{4}} \; \frac{9}{4}.$

$\because \; rt - s^2 > 0$

$\therefore$ There exists extreme values.

$\because \; r < 0$

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This implies that the above function has maximum values at $(\pi/3, \pi/3)$.

$$\text{value} = \sin\frac{\pi}{3} + \sin\frac{\pi}{3} + \sin\frac{2\pi}{3}$$

$$= \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{2} \quad \text{Ans.}$$

Question-2 In a plane triangle, ABC find the maximum value of $\cos A \cos B \cos C$.

Let $f = \cos A \cos B \cos C$.

$\because \quad A+B+C = \pi$

$C = \pi - (A+B)$

$$f(A,B) = \cos A \cos B \cos(\pi - (A+B))$$

$$= -\cos A \cos B \cos(A+B).$$

$$\frac{\partial f}{\partial A} = \sin A \cos B \cos(A+B) + \cos A \cos B \sin(A+B)$$

$$= \cos B\,(\sin A \cos(A+B) + \cos A \sin(A+B)$$

$$\partial f/\partial A = \cos B \sin(2A+B) = 0 \quad \text{------} \; 1$$

$$\frac{\partial f}{\partial B} = \sin B \cos A \cos(A+B) + \cos A \cos B \sin(A+B)$$

$$\frac{\partial f}{\partial B} = \cos A \sin(A+2B) = 0 \quad \text{------} \; 2.$$

Now,

$$r = \frac{\partial^2 f}{\partial A^2} = 2\cos B \cos(2A+B)$$

$$t = \frac{\partial^2 f}{\partial B^2} = 2\cos A \cos(A+2B)$$

$$s = \frac{\partial^2 f}{\partial A \partial B} = \cos(2A+2B).$$

Now, From eq. 1. and. 2.

Let $\sin(2A+B) = 0$ in 1.

$\therefore \quad \cos B = 0$

$B = \pi/2.$

Now, From 2,

$\cos A \sin(A+\pi) = 0.$

$= -\sin A \cos A = 0.$

Now, if we consider either $\cos A = 0$ i.e which is not possible, since $A+B+C=\pi$ and this will give $C=0$, which is not possible.

And $\sin A = 0$ is also not possible.

Thus from 2.,

Let $\cos B = 0$

$\therefore \sin(A+2B) = 0.$

$A+2B = \pi.$ ---------- 3.

Now, From 1.

$2A+B = \cancel{0}\pi$ ---------- 4.

Now, on solving 1. and 2.

$A = \pi/3$

$B = \pi/3$.

Now, $rt - s^2 = \frac{3}{4} > 0$

$\therefore$ there exists extreme values.

$\therefore r < 0$

$\therefore$ It will give maximum values at $(\pi/3, \pi/3)$.

$\therefore$ values $= \frac{1}{8}$ Ans.

Question-3 Examine for extreme values for the function $\cos x + \cos y + \cos(x+y)$.