classmate
Date
Page

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Lecture-27

Multiple integral :-

$\iint f(x,y)\, dy\, dx$, $\iiint f(x,y,z)\, dz\, dy\, dx$.

Question-1 Prove that $\int_1^2\int_3^4 (xy + e^y)\, dy\, dx = \int_3^4\int_1^2 (xy + e^y)\, dx\, dy$

LHS :-

$$= \int_1^2 \left[\frac{xy^2}{2} + e^y\right]_3^4 dx = \int_1^2 \left(8x + e^4 - \frac{9x}{2} - e^3\right) dx.$$

$$= \int_1^2 \left(\frac{7x}{2} + e^4 - e^3\right) dx$$

$$= \left[\frac{7x^2}{4} + (e^4 - e^3)x\right]_1^2$$

$$= 7 + 2(e^4 - e^3) - \frac{7}{4} - (e^4 - e^3).$$

$$= \frac{21}{4} + e^4 - e^3$$

RHS :-

$$\int_3^4\int_1^2 (xy + e^y)\, dx\, dy = \int_3^4 \left(\frac{x^2 y}{2} + xe^y\right]_1^2 dy$$

$$= \int_3^4 \left[2y + 2e^y - \frac{y}{2} - e^y\right] dy = \int_3^4 \left(\frac{3y}{2} + e^y\right) dy.$$

$$= \left[\frac{3y^2}{4} + e^y\right]_3^4 = \frac{21}{4} + e^4 - e^3.$$

LHS = RHS.

Hence prove.

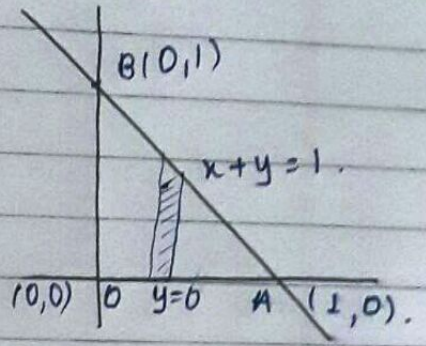
Question-2 Evaluate $\int_0^1\int_0^1 \frac{dx\, dy}{\sqrt{(1-x^2)(1-y^2)}}$

$$= \int_0^1 \left[\frac{1}{\sqrt{1-y^2}} \sin^{-1} x\right]_0^1 dy. = \int_0^1 \frac{1}{\sqrt{1-y^2}}\left(\frac{\pi}{2}\right) dy.$$

$= \frac{\pi}{2}\left[\sin^{-1} y\right]_0^{\pi/2} = \frac{\pi^2}{4}$ Ans.

Question-3 Evaluate $\iint e^{2x+3y}\,dx\,dy$ over the triangle bounded by $x=0$, $y=0$ and $x+y=1$

$= \int_0^1\int_0^{1-x} e^{2x+3y}\,dy\,dx$.



$= \int_0^1 e^{2x}\left(\frac{e^{3y}}{3}\right)_0^{1-x} dx$.

$= \int_0^1 \frac{e^{2x}}{3}(e^{3-3x}-1)\,dx$.

$= \int_0^1 \cancel{\frac{e^{3-x}}{3}-1}\,dx$. $= \frac{1}{3}\int_0^1 (e^3\cdot e^{-x} - e^{2x})\,dx$

$= \cancel{\frac{1}{3}\left[\frac{e^{3-x}}{-1}\right]_0^1}$ $= \frac{1}{3}\left[-e^3(e^{-x})_0^1 - \left(\frac{e^{2x}}{2}\right)_0^1\right]$.

$= \cancel{\frac{-1}{3}\cdot e^2}$ $= \frac{1}{3}\left[-e^3(e^{-1}-1) - \frac{1}{2}(e^2-1)\right]$

$= \frac{1}{3}\left(\frac{-e^3}{e}(1-e) - \frac{1}{2}(e+1)(e-1)\right]$.

$= \frac{1}{3}\left[e^2(e-1) - \frac{1}{2}(e+1)(e-1)\right]$.

$= \frac{1}{6}(e-1)\left[2e^2 - (e+1)\right]$

$= \frac{1}{6}(e-1)\left[2e^2 - 2e + e - 1\right]$

$= \frac{1}{6}(e-1)\left[2e(e-1) + 1(e-1)\right]$

$= \frac{1}{6}(e-1)(e-1)(2e+1)$

$= \frac{1}{6}(e-1)^2(2e+1)$. Ans.