Change of order of integration.

$$\int_c^d \int_a^b f(x,y)\,dx\,dy = \int_a^b \int_c^d f(x,y)\,dy\,dx.$$

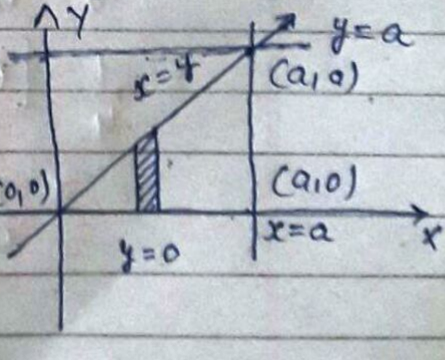
$$\int_a^b \int_{\phi_1(y)}^{\phi_2(y)} f(x,y)\,dx\,dy = \iint f(x,y)\,dy\,dx.$$

$\phi_1(y)$ and $\phi_2(y)$ are variable
a, b, c & d are constant.

Q-1 Change the order of integration in $\int_0^a \int_y^a \frac{x\,dx\,dy}{x^2+y^2}$ and hence evaluate the same.

Solution:-

$$\int_0^a \int_y^a \frac{x\,dx\,dy}{x^2+y^2} = \int_0^a \int_0^x \frac{x\,dy\,dx}{x^2+y^2}$$



$$= \int_0^a \int_0^x \frac{x}{x^2+y^2}\,dy\,dx.$$

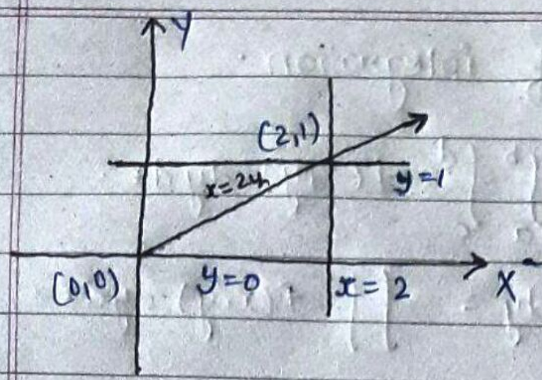
$$= \int_0^a \frac{1 \times x}{x}\left[\tan^{-1}\frac{y}{x}\right]_0^x dx$$

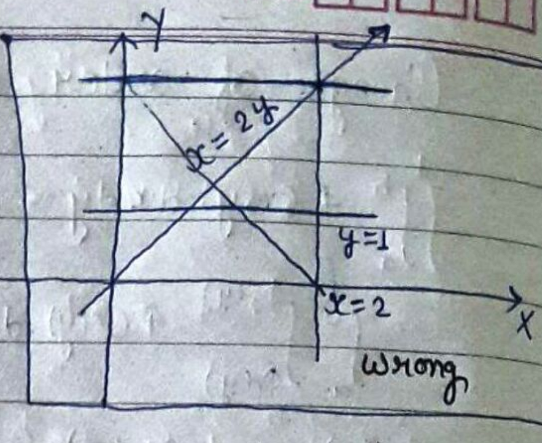
$$= \int_0^a \left[\tan^{-1}1 - \tan^{-1}0\right] dx$$

$$= \int_0^a \frac{\pi}{4}\,dx = \frac{\pi}{4}[x]_0^a = \frac{\pi a}{4} \quad \text{Ans}$$

Q-2 Evaluate by changing the order of integration $\int_0^1 \int_{2y}^2 e^{x^2}\,dx\,dy$.

Solution:- $\int_0^1 \int_{2y}^2 e^{x^2}\,dx\,dy = \int_0^2 \int_0^{x/2} e^{x^2}\,dy\,dx$





$$= \int_0^2 \int_0^{x/2} e^{x^2}\, dy\, dx$$

$$= \int_0^2 e^{x^2} \left[y \right]_0^{x/2} dx$$

$$= \int_0^2 e^{x^2} \frac{x}{2}\, dx$$

$$= \int_0^2 \frac{1}{4} e^t\, dt$$

Let $x^2 = t$

$\frac{dt}{2} = x\,dx$

$$= \frac{1}{4} \left[e^t \right]_0^4 = \frac{1}{4} \left[e^4 - e^0 \right]$$

$t = 4,\ t = 0$

$$= \frac{1}{4} (e^4 - 1) \quad \underline{\text{Ans}}$$

Q-3 Evaluate $\int_0^2 \int_1^{e^x} dy\, dx$, by changing the order of integration.

Solution:-

$$\int_0^2 \int_1^{e^x} dy\, dx = \int_1^{e^2} \int_{\log y}^{2} dx\, dy$$

$$= \int_1^{e^2} \left[x \right]_{\log y}^{2} dy$$

$$= \int_1^{e^2} (2 - \log y)\, dy$$

$$= e^2 - 3$$