13.02.23

Lecture-4

Composite Function:-

$y = \psi(t)$

- If $u = f(x, y)$, where $x = q(t)$, ~~$y = q'(t)$~~
u is called composite function of t
we can find $\frac{du}{dt}$

$$\frac{du}{dt} = \frac{\partial u}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial u}{\partial y} \cdot \frac{dy}{dt}$$

$$u = \frac{\partial u}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial u}{\partial y} \cdot \frac{dy}{dt}$$

$\frac{du}{dt}$ is called the total derivative of u.

- If $z = f(x, y)$, where $x = q(u, v)$, $y = \psi(u, v)$.
then z is called a composite function of (two variables) u and v, and we can find $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$

$$\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial u} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial u}$$

$$\frac{\partial z}{\partial v} = \frac{\partial z}{\partial z} \cdot \frac{\partial x}{\partial v} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial v}$$

- If $u = f(x, y, z)$, where x, y and z are the function of t, then we can find $\frac{du}{dt}$.

$$\frac{du}{dt} = \frac{\partial u}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial u}{\partial y} \cdot \frac{dy}{dt} + \frac{\partial u}{\partial z} \cdot \frac{dz}{dt}$$

- If $u = f(x, y)$ and $y = q(x)$ then

$$\frac{du}{dx} = \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} \cdot \frac{dy}{dx}.$$

- If we are given an implicit function $f(x, y) = c$
then $u = f(x, y)$, where $u = c$.

$$\frac{du}{dx} = \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} \cdot \frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{-\partial u / \partial x}{\partial u / \partial y}.$$

Question-1 If $u = x \log xy$, where $x^3 + y^3 + 3xy = 1$ find $\frac{du}{dx}$.

$$x^3 + y^3 + 3xy = 1.$$

$$= 3x^2 + 3y^2 \frac{dy}{dx} + 3x \frac{dy}{dx} + 3y = 0.$$

$$= \frac{dy}{dx} = \frac{-(x^2+y)}{(y^2+x)}.$$

$$\frac{\partial u}{\partial x} = x \frac{1}{xy} \cdot y + \log xy.$$

$$= 1 + \log xy.$$

$$\frac{\partial u}{\partial y} = x \frac{1}{xy} \cdot x.$$

$$\frac{\partial u}{\partial y} = \frac{x}{y}$$

Now, $$\frac{du}{dx} = \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} \cdot \frac{dy}{dx}$$

$$= 1 + \log xy + \frac{x}{y}\left(\frac{-(x^2+y)}{y^2+x}\right).$$

$$= 1 + \log xy - \frac{x(x^2+y)}{(y^2+x)y}$$. Ans.