Big-M Method (Method of Penalities)

The Big-M Method is an and alternative method of solving a linear LP involving artificial variable. In this method, we assign a very high penality (say M) to the artificial variable in the objective function

The iterated procedure of the algorithm is given below :-

1. We write the given LPP into its standard form and check whether there exist a starting basic feasible solution

a. If there is a ready starting basic feasible solution move on to step 4.

b. If there does not exist a ready starting basic feasible solution move on to step 2.

. Add artificial variable to the left side of each equation that has no obviows starting basic variables.
Assign a very high penality (say M) to these variables in the objective function

Apply simplex method to the modified LPP following cases arise at the last iteration

a. Atleast one artificial variable is present in the basis with 0 value. In such a case the current optimum basic feasible solution is degenerated.

b. Atleast one artifical variable is present in the basis with a +ve value. In such a case the given LPP does not possess an optimum basic feasible Soln. The given problem is said to pseudo optimum basic feasible solution.

Ques Use Big-M Method to solve

$$\text{Max } z = 6x_1 + 4x_2$$

s.t

$$2x_1 + 3x_2 \le 30$$
$$3x_1 + 2x_2 \le 24$$
$$x_1 + x_2 \ge 3.$$
$$x_1, x_2 \ge 0$$

Is the solution unique? If not give two different solution.

$$2x_1 + 3x_2 + x_3 = 30$$
$$3x_1 + 2x_2 + x_4 = 24$$
$$x_1 + x_2 - x_5 + x_6 = 3$$

6 4 0 0 0 −M

B	C_B	x_B	y_1	y_2	y_3	y_4	y_5	y_6	Min
y_3	0	30	2	3	1	0	0	0	15
y_4	0	24	3	2	0	1	0	0	8
y_6	−M	3	1	1	0	0	−1	1	3 →
	$z = C_B x_B = -3M$		−M−6 ↑	−M−4	0	0	M	0	

B	C_B	x_B	6 y_1	4 y_2	0 y_3	0 y_4	0 y_5	y_6	Min
y_3	0	24	0	1	1	0	2		12
y_4	0	15	0	-1	0	1	$\boxed{3}$		5 →
y_1	6	3	1	1	0	0	-1		
		Z=18	0	2	0	0	-6 ↑		

y_3	0	14	0	5/3	1	-2/3	0
y_5	0	5	0	-1/3	0	1/3	1
y_1	6	8	1	2/3	0	1/3	0
		Z=48	0	2	0	2	0

$x_1 = 8, \ x_2 = 0, \ \text{Max } Z = 48$

It is evident from the net evaluation of the optimum table that the net evaluation corresponding to non-basic variable $x_2 = 0$. This indicate that the current solution is not unique and alternative sol^n exist.

B	C_B	x_B	y_1	y_2	y_3	y_4	y_5	Min ratio
y_2	4	42/5	0	1	3/5	-2/5	0	
y_5	0	39/5	0	0	1/5	1/5	1	
y_1	6	12/5	1	0	-2/5	3/5	0	
		Z=48	0	0	0	2	0	

$x_1 = 12/5, \ x_2 = 42/5, \ \text{Max } Z = 48$

⇒ Alternative Solution