the indication of unbounded soln.

Ques

Use revised simplex method to solve the LPP

$$\max z = 3x_1 + 2x_2 + 5x_3$$

STC $x_1 + 2x_2 + x_3 \le 430$

$3x_1 + 2x_3 \le 460$

$x_1 + 4x_2 \le 420$

$x_1, x_2, x_3 \ge 0$

Soln

$$A = \begin{bmatrix} 1 & 2 & 1 & 1 & 0 & 0 \\ 3 & 0 & 2 & 0 & 1 & 0 \\ 1 & 4 & 0 & 0 & 0 & 1 \end{bmatrix} \rightarrow a_3, \quad b = \begin{bmatrix} 430 \\ 460 \\ 420 \end{bmatrix}$$

$$C^T = [3 \; 2 \; 5 \; 0 \; 0 \; 0]$$

$$\hat{A} = \begin{bmatrix} A \\ -C^T \end{bmatrix}$$

$$C_B^T = [0 \; 0 \; 0]$$

$$\hat{A} = \begin{bmatrix} 1 & 2 & 1 & 1 & 0 & 0 \\ 3 & 0 & 2 & 0 & 1 & 0 \\ 1 & 4 & 0 & 0 & 0 & 1 \\ -3 & -2 & -5 & 0 & 0 & 0 \end{bmatrix}$$

$$\hat{B}^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$z_j - c_j = [C_B^T B^{-1} \;\; 1] \hat{A}$$

$$= [0 \; 0 \; 0 \; 1] \begin{bmatrix} 1 & 2 & 1 & 1 & 0 & 0 \\ 3 & 0 & 2 & 0 & 1 & 0 \\ 1 & 4 & 0 & 0 & 0 & 1 \\ -3 & -2 & -5 & 0 & 0 & 2 \end{bmatrix}$$

$z_j - c_j = [-3 \quad -2 \quad -5 \quad 0 \quad 0 \quad 0]$
↑

Since $z_3 - c_3$ is almost -ve, therefore x_3 enters the basis

$$\hat{x}_3 = \hat{B}^{-1} a_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 0 \\ -5 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 0 \\ -5 \end{bmatrix}$$

$$\hat{x}_B = \hat{B}^{-1}_{current} \hat{b}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 430 \\ 460 \\ 420 \\ 0 \end{bmatrix} = \begin{bmatrix} 430 \\ 460 \\ 420 \\ 0 \end{bmatrix}$$

Initial revised simplex table

B	C_B	$\hat{X}_B$	$\hat{B}^{-1}$				$\hat{x}_3$	min
x_4	0	430	1	0	0	0	1	430
x_5		460	0	1	0	0	2	230 →
x_6		460	0	0	1	0	0	–
		0	0	0	0	1	-5	
							↑	

$$\hat{B}^{-1}_{current} = \begin{bmatrix} 1 & -1/2 & 0 & 0 \\ 0 & 1/2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 5/2 & 0 & 1 \end{bmatrix}$$

$$\hat{x}_B = \hat{B}^{-1} \hat{b} = \begin{bmatrix} 1 & -1/2 & 0 & 0 \\ 0 & 1/2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 5/2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 430 \\ 460 \\ 420 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 200 \\ 230 \\ 420 \\ 1150 \end{bmatrix}$$

$5 \times 230 = 1150$

$z_j - c_j = [C_B^T B^{-1} \quad 1] \hat{A}$

$B^{-1} = \begin{bmatrix} 0 & 1/2 \\ 0 & 0 & 1 \end{bmatrix}$

$$= [0 \; 5/2 \; 0 \; 1] \begin{bmatrix} 1 & 2 & 1 & 1 & 0 & 0 \\ 3 & 0 & 2 & 0 & 1 & 0 \\ 1 & 4 & 0 & 0 & 0 & 1 \\ -3 & -2 & -5 & 0 & 0 & 0 \end{bmatrix}$$

$$= [\,9/2 \;\; -2 \;\; 0 \;\; 0 \;\; 5/2 \;\; 0\,]$$

first revised simplex table:

$$\hat{x}_0 = \hat{B}^{-1}_{m} a_0$$

$$= \begin{bmatrix} 1 & -1/2 & 0 & 0 \\ 0 & 1/2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 5/2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \\ 4 \\ -2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ 0 \\ 4 \\ -2 \end{bmatrix}$$

B	$\hat{x}_B$	$\hat{B}^{-1}_m$				$\hat{x}_0$	min ratio
x_4	200	1	-1/2	0	0	2	100 →
x_2	230	0	1/2	0	0	0	-
x_6	420	0	0	1	0	4	105
	1150	0	5/2	0	1	-2 ↑	-

$$\hat{B}^{-1}_{m+1} = \begin{bmatrix} 1/2 & -1/4 & 0 & 0 \\ 0 & 1/2 & 0 & 0 \\ -2 & 1 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{bmatrix}$$

$$\hat{x}_B = \begin{bmatrix} 100 \\ 230 \\ 20 \\ 1350 \end{bmatrix}$$

$\frac{920-115}{...}$

$$C_B' B^{-1} = [2 \;\; 5 \;\; 0] \begin{bmatrix} 1/2 & -1/4 & 0 \\ 0 & 1/2 & 0 \\ -2 & 1 & 1 \end{bmatrix} = [1 \;\; 2 \;\; 0]$$

$z_j - c_j = [C_B^T B^{-1} \quad 1] \hat{A}$

$$= \begin{bmatrix} 1 & 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 & 1 & 0 & 0 \\ 3 & 0 & 2 & 0 & 1 & 0 \\ 1 & 4 & 0 & 0 & 0 & 1 \\ -3 & -2 & -5 & 0 & 0 & 0 \end{bmatrix}$$

$$= [4 \quad 0 \quad 0 \quad 1 \quad 2 \quad 0]$$

Since all $z_j - c_j \geq 0$, therefore the optimal basic feasible solⁿ is attained

ie $\hat{x}_B = \hat{B}^{-1}_{new} \hat{b}$

$$= \begin{bmatrix} 100 \\ 230 \\ 665 \\ 1350 \end{bmatrix}$$

Hence $x_1 = 100$

$x_2 = 230$

$x_3 = 665$

$\max x = 1350$